7

ISLAMIC UNIVERSITY OF TECHNOLOGY (IUT)
ORGANISATION OF ISLAMIC COOPERATION (OIC)

DEPARTMENT OF ELECTRICAL AND ELECTRONIC ENGINEERING

Mid-Semester Examination — Summer Semester, A.Y. 2022-2023
Course No.: EEE 4407 — Time: 90 Minutes
Course Title: Random Signals and Processes — Full Marks: 75

There are **3 (three)** questions. Answer **all** the questions. All questions carry equal marks. Marks in the margin indicate full marks. Programmable calculators are not allowed. Do not write on this question paper.

---

1. a) Define sample space and event space. Find their differences with appropriate example. 05 (CO1, PO1)

   b) In an experiment, *A*, *B*, *C*, and *D* are events with probabilities $P[A \cup B] = 5/8$, $P[A] = 3/8$, $P[C \cap D] = 1/3$, and $P[C] = 1/2$. Furthermore, *A* and *B* are disjoint, while *C* and *D* are independent. 10 (CO1, PO1)

   i) Find $P[A \cap B]$, $P[B]$, $P[A \cap B^C]$, and $P[A \cup B^C]$.
   ii) Are *A* and *B* independent?
   iii) Find $P[D]$, $P[C \cap D^C]$, $P[C^C \cap D^C]$, and $P[C|D]$.
   iv) Find $P[C \cup D]$ and $P[C \cup D^C]$.

   c) The time between telephone calls at a telephone switch is an exponential random variable T with expected value 0.01. Given $T > 0.02$, Find $E[T|T > 0.02]$, the conditional expected value of T. 10 (CO2, PO2)

2. a) Find the expected value of Geometric random variable. 05 (CO2, PO2)

   b) Suppose each day (starting on day 1) you buy one lottery ticket with probability *1/2*; otherwise, you buy no tickets. A ticket is a winner with probability p independent of the outcome of all other tickets. Let N_i be the event that on day *i* you do not buy a ticket. Let W_i be the event that on day *i*, you buy a winning ticket. Let L_i be the event that on day *i* you buy a losing ticket. 10 (CO2, PO2)

   i) Find P[W33], P[L87], and $P[N_{99}]$.
   ii) Let *K* be the number of the day on which you buy your first lottery ticket. Find the PMF $P_K(k)$.
   iii) Find the PMF of *R*, the number of losing lottery tickets you have purchased in *m* days.
   iv) Let *D* be the number of the day on which you buy your jth losing ticket. Find $P_D(d)$.

   c) The binomial random variable X has PMF 10 (CO2, PO2)

$$P_X(X) = \binom{5}{x}(1/2)^5.$$

i) Find the standard deviation of the random variable X.
ii) Find $P[\mu_X - \sigma_X \leq X \leq \mu_X + \sigma_X]$, the probability that X is within one standard deviation of the expected value?

3. a) Find PDF equation for exponential random variable. Find CDF, E[X] and Var [X] from this expression. 05 (CO2, PO2)

b) The peak temperature T, in degree Fahrenheit, on a July day in Antarctica is a Gaussian random variable with a variance of 225. With probability $1/2$, the temperature T exceeds 10 degrees. Find $P[T > 32]$, the probability the temperature is above freezing. Find $P[T < 0]$ and $P[T > 60]$. 10 (CO2, PO2)

c) X is a continuous uniform $(-5, 5)$ random variable. 10 (CO2, PO2)

i) Find PDF $f_X(x)$.
ii) Find CDF $F_X(x)$.
iii) Find $E[X]$.
iv) Find $E[X^5]$.
v) Find $E[e^X]$.

CDF chart for standard normal distribution

z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.5000	0.5040	0.5080	0.5120	0.5160	0.5199	0.5239	0.5279	0.5319	0.5359
0.1	0.5398	0.5438	0.5478	0.5517	0.5557	0.5596	0.5636	0.5675	0.5714	0.5753
0.2	0.5793	0.5832	0.5871	0.5910	0.5948	0.5987	0.6026	0.6064	0.6103	0.6141
0.3	0.6179	0.6217	0.6255	0.6293	0.6331	0.6368	0.6406	0.6443	0.6480	0.6517
0.4	0.6554	0.6591	0.6628	0.6664	0.6700	0.6736	0.6772	0.6808	0.6844	0.6879
0.5	0.6915	0.6950	0.6985	0.7019	0.7054	0.7088	0.7123	0.7157	0.7190	0.7224
0.6	0.7257	0.7291	0.7324	0.7357	0.7389	0.7422	0.7454	0.7486	0.7517	0.7549
0.7	0.7580	0.7611	0.7642	0.7673	0.7703	0.7734	0.7764	0.7794	0.7823	0.7852
0.8	0.7881	0.7910	0.7939	0.7967	0.7995	0.8023	0.8051	0.8078	0.8106	0.8133
0.9	0.8159	0.8186	0.8212	0.8238	0.8264	0.8289	0.8315	0.8340	0.8365	0.8389
1.0	0.8413	0.8438	0.8461	0.8485	0.8508	0.8531	0.8554	0.8577	0.8599	0.8621
1.1	0.8643	0.8665	0.8686	0.8708	0.8729	0.8749	0.8770	0.8790	0.8810	0.8830
1.2	0.8849	0.8869	0.8888	0.8907	0.8925	0.8944	0.8962	0.8980	0.8997	0.90147
1.3	0.90320	0.90490	0.90658	0.90824	0.90988	0.91149	0.91309	0.91466	0.91621	0.91774
1.4	0.91924	0.92073	0.92220	0.92364	0.92507	0.92647	0.92785	0.92922	0.93056	0.93189
1.5	0.93319	0.93448	0.93574	0.93699	0.93822	0.93943	0.94062	0.94179	0.94295	0.94408
1.6	0.94520	0.94630	0.94738	0.94845	0.94950	0.95053	0.95154	0.95254	0.95352	0.95449
1.7	0.95543	0.95637	0.95728	0.95818	0.95907	0.95994	0.96080	0.96164	0.96246	0.96327
1.8	0.96407	0.96485	0.96562	0.96638	0.96712	0.96784	0.96856	0.96926	0.96995	0.97062
1.9	0.97128	0.97193	0.97257	0.97320	0.97381	0.97441	0.97500	0.97558	0.97615	0.97670
2.0	0.97725	0.97778	0.97831	0.97882	0.97932	0.97982	0.98030	0.98077	0.98124	0.98169
2.1	0.98214	0.98257	0.98300	0.98341	0.98382	0.98422	0.98461	0.98500	0.98537	0.98574
2.2	0.98610	0.98645	0.98679	0.98713	0.98745	0.98778	0.98809	0.98840	0.98870	0.98899
2.3	0.98928	0.98956	0.98983	$0.9^{2}0097$	$0.9^{2}0358$	$0.9^{2}0613$	$0.9^{2}0863$	$0.9^{2}1106$	$0.9^{2}1344$	$0.9^{2}1576$
2.4	$0.9^{2}1802$	$0.9^{2}2024$	$0.9^{2}2240$	$0.9^{2}2451$	$0.9^{2}2656$	$0.9^{2}2857$	$0.9^{2}3053$	$0.9^{2}3244$	$0.9^{2}3431$	$0.9^{2}3613$
2.5	$0.9^{2}3790$	$0.9^{2}3963$	$0.9^{2}4132$	$0.9^{2}4297$	$0.9^{2}4457$	$0.9^{2}4614$	$0.9^{2}4766$	$0.9^{2}4915$	$0.9^{2}5060$	$0.9^{2}5201$
2.6	$0.9^{2}5339$	$0.9^{2}5473$	$0.9^{2}5604$	$0.9^{2}5731$	$0.9^{2}5855$	$0.9^{2}5975$	$0.9^{2}6093$	$0.9^{2}6207$	$0.9^{2}6319$	$0.9^{2}6427$
2.7	$0.9^{2}6533$	$0.9^{2}6636$	$0.9^{2}6736$	$0.9^{2}6833$	$0.9^{2}6928$	$0.9^{2}7020$	$0.9^{2}7110$	$0.9^{2}7197$	$0.9^{2}7282$	$0.9^{2}7365$
2.8	$0.9^{2}7445$	$0.9^{2}7523$	$0.9^{2}7599$	$0.9^{2}7673$	$0.9^{2}7744$	$0.9^{2}7814$	$0.9^{2}7882$	$0.9^{2}7948$	$0.9^{2}8012$	$0.9^{2}8074$
2.9	$0.9^{2}8134$	$0.9^{2}8193$	$0.9^{2}8250$	$0.9^{2}8305$	$0.9^{2}8359$	$0.9^{2}8411$	$0.9^{2}8462$	$0.9^{2}8511$	$0.9^{2}8559$	$0.9^{2}8605$
3.0	$0.9^{2}8650$	$0.9^{2}8694$	$0.9^{2}8736$	$0.9^{2}8777$	$0.9^{2}8817$	$0.9^{2}8856$	$0.9^{2}8893$	$0.9^{2}8930$	$0.9^{2}8965$	$0.9^{2}8999$
3.1	$0.9^{3}0324$	$0.9^{3}0646$	$0.9^{3}0957$	$0.9^{3}1260$	$0.9^{3}1553$	$0.9^{3}1836$	$0.9^{3}2112$	$0.9^{3}2378$	$0.9^{3}2636$	$0.9^{3}2886$
3.2	$0.9^{3}3129$	$0.9^{3}3363$	$0.9^{3}3590$	$0.9^{3}3810$	$0.9^{3}4024$	$0.9^{3}4230$	$0.9^{3}4429$	$0.9^{3}4623$	$0.9^{3}4810$	$0.9^{3}4991$
3.3	$0.9^{3}5166$	$0.9^{3}5335$	$0.9^{3}5499$	$0.9^{3}5658$	$0.9^{3}5811$	$0.9^{3}5959$	$0.9^{3}6103$	$0.9^{3}6242$	$0.9^{3}6376$	$0.9^{3}6505$
3.4	$0.9^{3}6631$	$0.9^{3}6752$	$0.9^{3}6869$	$0.9^{3}6982$	$0.9^{3}7091$	$0.9^{3}7197$	$0.9^{3}7299$	$0.9^{3}7398$	$0.9^{3}7493$	$0.9^{3}7585$
3.5	$0.9^{3}7674$	$0.9^{3}7759$	$0.9^{3}7842$	$0.9^{3}7922$	$0.9^{3}7999$	$0.9^{3}8074$	$0.9^{3}8146$	$0.9^{3}8215$	$0.9^{3}8282$	$0.9^{3}8347$
3.6	$0.9^{3}8409$	$0.9^{3}8469$	$0.9^{3}8527$	$0.9^{3}8583$	$0.9^{3}8637$	$0.9^{3}8689$	$0.9^{3}8739$	$0.9^{3}8787$	$0.9^{3}8834$	$0.9^{3}8879$
3.7	$0.9^{3}8922$	$0.9^{3}8964$	$0.9^{4}0039$	$0.9^{4}0426$	$0.9^{4}0799$	$0.9^{4}1158$	$0.9^{4}1504$	$0.9^{4}1838$	$0.9^{4}2159$	$0.9^{4}2568$
3.8	$0.9^{4}2765$	$0.9^{4}3052$	$0.9^{4}3327$	$0.9^{4}3593$	$0.9^{4}3848$	$0.9^{4}4094$	$0.9^{4}4331$	$0.9^{4}4558$	$0.9^{4}4777$	$0.9^{4}4988$
3.9	$0.9^{4}5190$	$0.9^{4}5385$	$0.9^{4}5573$	$0.9^{4}5753$	$0.9^{4}5926$	$0.9^{4}6092$	$0.9^{4}6253$	$0.9^{4}6406$	$0.9^{4}6554$	$0.9^{4}6696$
4.0	$0.9^{4}6833$	$0.9^{4}6964$	$0.9^{4}7090$	$0.9^{4}7211$	$0.9^{4}7327$	$0.9^{4}7439$	$0.9^{4}7546$	$0.9^{4}7649$	$0.9^{4}7748$	$0.9^{4}7843$
4.1	$0.9^{4}7934$	$0.9^{4}8022$	$0.9^{4}8106$	$0.9^{4}8186$	$0.9^{4}8263$	$0.9^{4}8338$	$0.9^{4}8409$	$0.9^{4}8477$	$0.9^{4}8542$	$0.9^{4}8605$
4.2	$0.9^{4}8665$	$0.9^{4}8723$	$0.9^{4}8778$	$0.9^{4}8832$	$0.9^{4}8882$	$0.9^{4}8931$	$0.9^{4}8978$	$0.9^{5}0226$	$0.9^{5}0655$	$0.9^{5}1066$
4.3	$0.9^{5}1460$	$0.9^{5}1837$	$0.9^{5}2199$	$0.9^{5}2545$	$0.9^{5}2876$	$0.9^{5}3193$	$0.9^{5}3497$	$0.9^{5}3788$	$0.9^{5}4066$	$0.9^{5}4332$
4.4	$0.9^{5}4587$	$0.9^{5}4831$	$0.9^{5}5065$	$0.9^{5}5288$	$0.9^{5}5502$	$0.9^{5}5706$	$0.9^{5}5902$	$0.9^{5}6089$	$0.9^{5}6268$	$0.9^{5}6439$
4.5	$0.9^{5}6602$	$0.9^{5}6759$	$0.9^{5}6908$	$0.9^{5}7051$	$0.9^{5}7187$	$0.9^{5}7318$	$0.9^{5}7442$	$0.9^{5}7561$	$0.9^{5}7675$	$0.9^{5}7784$
4.6	$0.9^{5}7888$	$0.9^{5}7987$	$0.9^{5}8081$	$0.9^{5}8172$	$0.9^{5}8258$	$0.9^{5}8340$	$0.9^{5}8419$	$0.9^{5}8494$	$0.9^{5}8566$	$0.9^{5}8634$
4.7	$0.9^{5}8699$	$0.9^{5}8761$	$0.9^{5}8821$	$0.9^{5}8877$	$0.9^{5}8931$	$0.9^{5}8983$	$0.9^{6}0320$	$0.9^{6}0789$	$0.9^{6}1235$	$0.9^{6}1661$
4.8	$0.9^{6}2067$	$0.9^{6}2453$	$0.9^{6}2822$	$0.9^{6}3173$	$0.9^{6}3508$	$0.9^{6}3827$	$0.9^{6}4131$	$0.9^{6}4420$	$0.9^{6}4696$	$0.9^{6}4958$
4.9	$0.9^{6}5208$	$0.9^{6}5446$	$0.9^{6}5673$	$0.9^{6}5889$	$0.9^{6}6094$	$0.9^{6}6289$	$0.9^{6}6475$	$0.9^{6}6652$	$0.9^{6}6821$	$0.9^{6}6981$

Example: $\Phi(3.39) = 0.9996505$, $\Phi(0.98) = 0.8365$.