

ISLAMIC UNIVERSITY OF TECHNOLOGY (IUT)
ORGANISATION OF ISLAMIC COOPERATION (OIC)
DEPARTMENT OF NATURAL SCIENCES

Mid Semester Examination

Summer Semester, A.Y. 2022 - 2023

Course Number: Math 4211

Full Marks: 75

Course Title: PDE, Special Functions, Laplace and
Fourier Analysis

Time : 1.5 Hours

There are 3 (three) questions. Answer all questions. The symbols have their usual meanings. Marks of each question and corresponding CO and PO are written in the brackets.

1. a) Determine the interval of convergence and radius of convergence of the power series $f(x) = \sum_{n=1}^{\infty} \frac{(n+1)}{(n+2)(n+3)} x^n$. [10] CO1 PO1
- b) A differential equation is given below: $4xy'' + 2y' + y = 0$. CO1 PO1
(i) Determine and classify the singular points of the above differential equation. [5]
(ii) Find the power series solution of the above given differential equation. [10]
2. a) (i) Given a function $f(t)$ graphed by the Fig. Q2(a), find the Laplace transform of $f(t)$. [8] CO2 PO1

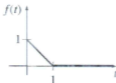


Fig.Q2(a)

- (ii) Compute the following: [7] CO2 PO1
- $$L^{-1} \left\{ \frac{8s^2 - 4s + 12}{s(s^2 + 4)} \right\}$$
- b) Find the solution of the following differential equation along with initial conditions using Laplace transform. [10] CO2 PO1

$$y'' + y = \sin(2t); y(0) = 2, y'(0) = 1.$$

3. a) (i) Expand $f(x) = x + x^2$, $-\pi < x < \pi$ in Fourier series [10] CO2
PO1
- (ii) Hence show that $\frac{\pi^2}{6} = 1 + \frac{1}{2^2} + \frac{1}{3^2} + \frac{1}{4^2} + \frac{1}{5^2} + \dots$ [5]
- b) Find the Fourier half range cosine series of $f(x) = \begin{cases} 2x, & 0 < x < 1 \\ 2(2-x), & 1 < x < 2 \end{cases}$ [10] CO2
PO1

The End