

ISLAMIC UNIVERSITY OF TECHNOLOGY (IUT)
ORGANISATION OF ISLAMIC COOPERATION (OIC)

DEPARTMENT OF NATURAL SCIENCES

Mid Semester Examination

Summer Semester, A. Y. 2022-2023

Course No.: Math 4221/Math 4629

Time: 90 minutes

Course Title: Mathematics III

Full Marks: 75

There are 3 (three) questions. Answer all 3 (three) questions. The symbols have their usual meanings. Programmable calculators are not allowed. Marks of each question and corresponding COs and POs are written in the brackets.

1. a) Explain the physical significance of dot and cross product of two vectors. (12)
(CO1)
(PO1)
- b) Show that $\nabla \cdot (\nabla \times \mathbf{A}) = 0$. (6)
(CO1)
(PO1)
- c) Find ϕ if $\nabla \phi = (y^2 - 2xyz^2)\mathbf{i} + (3 + 2xy - x^2z^2)\mathbf{j} + (6z^3 - 3x^2yz^2)\mathbf{k}$. (7)
(CO1)
(PO1)
2. a) Show that ∇f is a vector perpendicular to the level surface $f(x, y, z) = c$ where c is constant. Find the unit normal to the surface $3x^2 - y^2 + 2z = 1$ at $(1, 2, 1)$. (12)
(CO1)
(PO1)
- b) If $\mathbf{A} = (3x+y)\mathbf{i} - x\mathbf{j} + (y-2)\mathbf{k}$ and $\mathbf{B} = 2\mathbf{i} - 3\mathbf{j} + \mathbf{k}$ evaluate $\int_C (\mathbf{A} \times \mathbf{B}) \cdot d\mathbf{r}$ around the circle (13)
(CO2)
(PO1)
in the xy -plane having center at the origin and radius 2, traversed in the positive direction.
3. a) Find the flow of current through the surface S where S is the part of the plane $2x + 3y + 6z = 12$ which is located in the first octant, if the current density is $\mathbf{J} = 10z\mathbf{i} + 10\mathbf{j} + 3y\mathbf{k}$. (12)
(CO3)
(PO1)
- b) State and prove Gauss divergence theorem. (13)
(CO2)
(PO1)