

5

ISLAMIC UNIVERSITY OF TECHNOLOGY (IUT)
ORGANISATION OF ISLAMIC COOPERATION (OIC)

DEPARTMENT OF NATURAL SCIENCES

Mid Semester Examination

Course Number: Math 4241

Course Title: Integral Calculus and Differential Equations

Summer Semester, A.Y. 2022 - 2023

Full Marks: 100

Time : 1.5 Hours

Programmable calculators are not allowed. Do not write anything on the question paper. Answer all **3 (three)** questions. Figures in the right margin indicate full marks of questions whereas corresponding CO and PO are written within parentheses.

1. a) Define antidifferentiation with examples. (5)
(CO1)
(PO1)
- b) Find the value of (i) $\int \tan^4 x \sec x \, dx$, (ii) $\int_0^{\pi/2} \sin^5 x \, dx$ (12)
(CO2)
(PO2)
- c) Determine the value of $\int_0^{\pi/2} \log(\cos x) \, dx$ and hence compute $\int_0^{\pi/2} \frac{\log\left(x + \frac{1}{x}\right)}{1+x^2} \, dx$. (16)
(CO2)
(PO2)
2. a) Define Beta and Gamma function with examples. (6)
(CO1)
(PO1)
- b) Use the properties of Gamma function to find the value of $\int_0^1 \frac{x^2 \, dx}{\sqrt{1-x^4}} \times \int_0^1 \frac{dx}{\sqrt{1+x^4}}$. (12)
(CO2)
(PO2)
- c) Consider $\frac{dy}{dx} = -x^2 y$. Draw the direction field. Hence sketch an approximate solution curve when $y(1) = 1$. (8)
(CO2)
(PO2)
- d) Determine whether the following differential equation is exact or not. If not, make it exact. Then solve the transformed equation. (8)
(CO2)
(PO2)
$$x \, dx + (x^2 y + 4y) \, dy = 0, \quad y(4) = 0.$$
3. a) Determine the solution of (9)
(CO2)
(PO2)
$$-y \, dx + (x + \sqrt{xy}) \, dy = 0$$
 by using an appropriate substitution.
- b) Solve the initial value problem $x^2 \frac{dy}{dx} - 2xy = 3y^4, \quad y(1) = 1/2$. (9)
(CO2)
(PO2)
- c) Find the solution of $\frac{d^2 y}{dx^2} - 6 \frac{dy}{dx} + 9y = 6x^2 + 2 - 12e^{3x}$ using an appropriate method. (15)
(CO2)
(PO2)