

**ISLAMIC UNIVERSITY OF TECHNOLOGY (IUT)**  
 ORGANISATION OF ISLAMIC COOPERATION (OIC)  
 DEPARTMENT OF NATURAL SCIENCES

MID SEMESTER EXAMINATION  
 COURSE NO.: Math 4253  
 COURSE TITLE: Vector Algebra, Vector  
 Calculus and ODE

SUMMER SEMESTER A. Y. 2022-2023  
 TIME: 1.5 Hours  
 FULL MARKS: 75

There are 3 (Three) questions. Answer 3 (Three) questions. Programmable calculators are not allowed. Do not write on this question paper. The figures in the right margin indicate full marks and corresponding CO and PO in the brackets. Symbols convey their usual meanings. Assume reasonable values for any missing data.

1. (a) Find the differential equation of the family of parabolas with foci at the origin and axes along  $x - axis$ . Sketch some family members of such parabolas. (8)  
(CO1)  
(PO1)
- (b) Solve the differential equation:  $2x \, dy - 2y \, dx = \sqrt{x^2 + 4y^2} \, dx$ . (8)  
(CO1)  
(PO1)
- (c) (i) What is integrating factor? (ii) Determine the integrating factor of the differential equation  $\frac{dy}{dx} + \frac{1}{3}y = \frac{1}{3}(1 - 2x)y^4$  and solve it using integrating factor. (2+7)  
(CO1)  
(PO1)
2. (a) (i) Find the orthogonal trajectories of the family of the curves  $x^2 + 2y^2 = C$ . (3+5)  
(ii) A certain radioactive material is known to decay at rate proportional to the amount present. If initially 500 mg of the material is present and after 3 years 20 percent of the original mass has decayed, find an expression for the mass at any time. (CO3)  
(PO2)
- (b) Examine whether the following three vectors  $\underline{A} = 2\hat{i} + \hat{j} - 3\hat{k}$ ,  $\underline{B} = \hat{i} - 4\hat{k}$  and  $\underline{C} = 4\hat{i} + 3\hat{j} - \hat{k}$  are linearly dependent or independent. Determine a relation between them and hence show that the terminal points are collinear. (8)  
(CO1)  
(PO1)
- (c) Solve the following simultaneous equations for  $\underline{x}$  and  $\underline{y}$ : (9)  
(CO3)  
(PO2)  

$$\underline{x} + \underline{y} = \underline{a}, \quad \underline{x} \times \underline{y} = \underline{b} \text{ and } \underline{x} \cdot \underline{a} = 1.$$
3. (a) A particle moves along the curve  $x = t^3 + 1, y = t^2, z = 2t + 5$ , where  $t$  is the time. Find the components of its velocity and acceleration at  $t = 1$  (9)  
(CO1)

in the direction  $2\hat{i} + 2\hat{j} + 6\hat{k}$ . (PO1)

(b) A force  $\underline{F} = 3\hat{i} + 2\hat{j} - 4\hat{k}$  is applied at the point  $(1, -1, 2)$ . Find the moment of the force about the point  $(2, -1, 3)$ . (4)  
(CO2)  
(PO2)

(c) If  $\underline{a}$ ,  $\underline{b}$ ,  $\underline{c}$  be three-unit vectors such that  $\underline{a} \times (\underline{b} \times \underline{c}) = \frac{1}{2}\underline{b}$ , find the angles which  $\underline{a}$  make with  $\underline{b}$  and  $\underline{c}$ ,  $\underline{b}$  and  $\underline{c}$  being non parallel. (4)  
(CO1)  
(PO1)

(d) Find the value of  $\underline{r}$  satisfying the equation  $\frac{d^2 \underline{r}}{dt^2} = \underline{a}$ , where  $\underline{a}$  is a constant vector. (8)  
(CO1)  
(PO1)  
Also is given that when  $t = 0$ ,  $\underline{r} = \underline{0}$  and  $\frac{d\underline{r}}{dt} = \underline{u}$