

ISLAMIC UNIVERSITY OF TECHNOLOGY (IUT)
ORGANISATION OF ISLAMIC COOPERATION (OIC)
DEPARTMENT OF NATURAL SCIENCES

Mid Semester Examination
Course No. : MATH 4261
Course Title : Mathematics II

Summer Semester, A. Y. 2022-2023
Time : 1.5 hours
Full Marks : 75

Answer all 3 (three) questions. All questions carry equal marks. Marks of each question and corresponding CO and PO are written in the right margin with brackets.

1. a) The solid waste generated each year in the cities of Bangladesh is increasing. The solid waste generated, in millions of tons, was 238.3 in 2015 and 251.3 in 2021. (5) (CO1) (PO1)

Assuming that the amount of solid waste generated by Bangladesh cities is a linear function of time, find a formula for this function by finding the equation of the line through these two points. Use this formula to find the amount of solid waste generated in the year 2025.

- b) Raggs, Ltd., a clothing firm, has fixed costs of \$10,000 per year. These costs, such as rent, maintenance, and so on, must be paid no matter how much the company produces. To produce x units of a certain kind of suit, it costs \$20 per suit (unit) in addition to the fixed costs. That is, the variable costs for producing x of these suits are dollars. These costs are due to the amount produced and stem from items such as material, wages, fuel, and so on. The total cost $C(x)$ of producing x suits in a year is given by a function C : (10) (CO3) (PO2)

$$C(x) = (\text{Variable costs}) + (\text{Fixed costs}) = 20x + 10,000.$$

Graph the variable-cost, the fixed-cost, and the total-cost functions. Determine the total cost of producing 100 suits and 400 suits?

- c) The Custom Office Company makes an executive line of desks. It is estimated that the total cost of making x units of their Senior Executive Model desk is $C(x) = 100x + 200,000$ dollars per year, so that the average cost of making x units of the desks is given by $A(x) = \frac{C(x)}{x} = 100 + \frac{200,000}{x}$ dollars per desk. Deduce $\lim_{x \rightarrow \infty} A(x)$ and interpret your results. (5) (CO1) (PO1)
- d) Marcy works at the B & O department store, where on a weekday, she is paid \$6 per hour for the first eight hours and \$9 per hour for overtime. You may verify that the (5) (CO2) (PO2)

function $f(x) = \begin{cases} 6x & ; 0 \leq x \leq 8 \\ 9x - 24 & ; 8 < x \end{cases}$ gives Marcy's earnings on a weekday in which she worked x hours. Discuss the continuity of the function at $x = 8$ using graph.

2. a) Find values of the constants k and m , if possible, that will make the function f continuous everywhere. (10) (CO2) (PO1)

$$f(x) = \begin{cases} x^2 + 5, & x > 2 \\ m(x+1) + k, & -1 < x \leq 2 \\ 2x^3 + x + 7, & x \leq -1 \end{cases}$$

- b) The demand function for the Sicard wrist watch is given by (8) (CO3) (PO2)

$$d(x) = \frac{50}{0.01x^2 + 1} \quad (0 \leq x \leq 20)$$

where x (thousand) is the quantity demanded per week and $d(x)$ is the unit price in dollars. Find $d'(x)$. Hence compute $d'(5)$, $d'(10)$ and $d'(15)$ and interpret your results.

- c) If $y = a \cos(\ln x) + b \sin(\ln x)$, then applying Leibnitz's Theorem find the value of $x^2 y_{n+2} + (2n+1)xy_{n+1} + (n^2+1)y_n$. (7) (CO1) (PO1)

3. a) Apply L' Hospital's Rule to evaluate the following limit (10) (CO3) (PO2)

$$\lim_{x \rightarrow 0} \left[\frac{1}{x^2} - \frac{1}{\sin^2 x} \right]$$

- b) The number of major crimes committed in the city of Dhaka from 2010 to 2017 is approximated by the function $N(t) = -0.1t^3 + 1.5t^2 + 100$, $0 \leq t \leq 7$; (6) (CO2) (PO2)

where $N(t)$ denotes the number of crimes committed in year t , with $t = 0$ corresponding to the beginning of 2010. Find where the function N is increasing and where it is decreasing using first derivative.

- c) If $V = \sqrt{x^2 + y^2 + z^2}$ then show that $V_{xx} + V_{yy} + V_{zz} = 2/V$. (9) (CO2) (PO1)