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B.Sc. Eng. (ME/IPE)/2nd Sem. **May 21, 2024**

ISLAMIC UNIVERSITY OF TECHNOLOGY (IUT)
ORGANISATION OF ISLAMIC COOPERATION (OIC)
DEPARTMENT OF NATURAL SCIENCES

Semester Final Examination — Summer Semester: 2022-2023
Course Number: Math 4211 — Full Marks: 150
Course Title: PDE, Special Functions, Laplace and Fourier Analysis — Time: 3 Hours

Answer all the **6 (Six)** questions. The symbols have their usual meanings. Marks of each question and the corresponding CO and PO are written in the brackets.

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1. a) (i) Write Bessel's differential equation of order n. **(2+13)** (CO1)
(ii) Derive Bessel's function of 1st kind from part a(i). (PO1)

b) Express $f(x) = 4x^3 + 6x^2 + 7x + 2$ in terms of Legendre Polynomials. **(10)** (CO1) (PO1)

2. a) Expand $f(x) = x; -2 < x < 2$, in a complex Fourier series. **(10)** (CO2) (PO1)

b) Apply finite Fourier transform to solve the one-dimensional heat conduction equation $\frac{\partial T}{\partial t} = \frac{\partial^2 T}{\partial x^2}$, subject to the boundary conditions in Fig. Q2(b) and the initial condition $T(x,0) = 2x$, where $0 < x < 4$. **(15)** (CO2) (PO2)

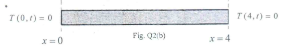

Fig. Q2(b)

3. a) Find the Fourier integrals of the function $f(x) = \begin{cases} 1, & |x| < a \\ 0, & |x| > a \end{cases}$ and hence evaluate the integral $\int_0^\infty \frac{\sin au \cos ux}{u} du$. **(10)** (CO2) (PO1)

b) Use Laplace Transform to find the solution of the following initial value problem: $y'' + 4y = g(t)$, $y(0) = 0$, $y'(0) = 0$, **(15)** (CO2) (PO2)

where $g(t)$ is the ramp loading as shown in Fig. Q3(b).

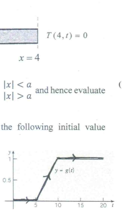

Fig. Q3(b)

4. a) Consider the following function: (10) (CO2) (PO1)

$$f(t)=\begin{cases}2; & 0<t<4\\ 5; & 4\le t<7\\ -1; & 7\le t<9\\ 1; & t\ge 9\end{cases}$$

Express f in terms of unit step functions and hence evaluate Laplace transform of the function.

b) A beam of length L is embedded at both ends, as shown in Fig. Q4(b). Find the deflection of the beam when the load is given by (15) (CO2) (PO2)

$$w(x)=\begin{cases}w_0\left(1-\frac{2}{L}x\right); & 0<x<\frac{L}{2}\\ 0; & \frac{L}{2}<x<L\end{cases}$$

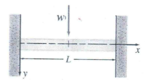

Fig. Q4(b)

5. a) Formulate partial differential equations by eliminating arbitrary constants a, b, and c from the following function: (10) (CO3) (PO1)

$$z=ax+by+cxy$$

where z is a dependent variable and x, y are independent variables.

b) Solve the following partial differential equation: (15) (CO3) (PO1)

$$\left(D^3-7DD'^2-6D'^3\right)z=e^{x+y}+x^3.$$

where $D=\frac{\partial}{\partial x}$ and $D'=\frac{\partial}{\partial y}$

6. a) In steady state conditions derive one dimensional heat flow equation. (10) (CO3) (PO2)

b) Solve the following wave equation using the method of separation of variables: (15) (CO2) (PO2)

$$\frac{\partial^2 y}{\partial t^2}=c^2\frac{\partial^2 y}{\partial x^2}$$

where $y=y(x,t)$ is the displacement of a tightly stretched string at time t with the following initial and boundary conditions:

$$y(0,t)=y(l,t)=0;\ l \text{ denotes the length of the string,}$$

$$y(x,0)=y_0\sin^3\left(\frac{\pi x}{l}\right) \text{ and } \frac{\partial y}{\partial t}=0 \text{ at } t=0.$$