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B.Sc. Eng. (EEE, 2nd Sem.)/B.Sc.TE (2Y, 2nd Sem.) **May 22, 2024**

ISLAMIC UNIVERSITY OF TECHNOLOGY (IUT)
ORGANISATION OF ISLAMIC COOPERATION (OIC)
DEPARTMENT OF NATURAL SCIENCES

Semester Final Examination
Course No.: Math 4221
Course Title: Mathematics III (Complex Variable, Vector Analysis and Statistics)

Summer Semester, A. Y. 2022-2023
Time: 3 Hours
Full Marks: 150

There are **6 (six)** questions. Answer all **6 (six)** questions. The symbols have their usual meanings. Programmable calculators are not allowed. Marks of each question and corresponding COs and POs are written in the brackets.

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1. (a) Give the physical interpretation of divergence and curl of a vector function. Determine the constant a so that vector **F** =(-6x-6y+3z)i+(-2x+4y-5z)j+(5x+6y+2az)k is solenoidal. **[12]** (CO1, PO1)

(b) Evaluate $\iint_S \varphi \mathbf{n} dS$ where $\varphi = xyz$ and S is the surface of the cylinder $x^2+y^2=16$ included in the first octant between z=0 and z=3. **[13]** (CO1, PO1)

2. (a) Verify Green's theorem in the plane $\oint_C (3x^2 - 8y^2)dx + (4y - 6xy)dy$, where C is the closed curve of the region bounded by $y = \sqrt{x}$ and $y = x^2$. **[12]** (CO2, PO1)

(b) Find all the roots of $(-4 + 4i)^{\frac{1}{7}}$ and locate them in the complex plane. **[13]** (CO2, PO1)

3. (a) Define conformal mapping. Show that the transformation $w = \frac{3-z}{z-2}$ transforms the circle with centre $\left(\frac{5}{2}, 0\right)$ and radius $\frac{1}{2}$ in the z-plane into the imaginary axis in the w-plane. **[12]** (CO2, PO1)

(b) Derive Cauchy- Riemann equations for $f(z) = u + iv$. Show that u and v satisfy Laplace equation. **[13]** (CO1, PO1)

4. (a) For the harmonic function $u = 3x^2y + 2x^2 - y^3 - 2y^2$ find its harmonic conjugate v and express $u + iv$ as an analytic function of z. Is the function $e^{-2xy} \sin(x^2 - y^2)$ harmonic? **[12]** (CO1, PO1)

(b) State and prove Cauchy's integral theorem. Evaluate $\int_C \frac{11z+13}{z^4+9z^3+20z^2}dz$ if the contour C is the circle $|z-3|=2$. [13] (CO1, (PO1)

5. (a) Find the residue of $f(z)=\frac{z^2-2z}{(z+1)^2(z^2+3z-4)}$ at all poles. [12] (CO1, PO1)

(b) Find the regression line of Y on X from the following data and find Y when X=3. [13] (CO3, PO1)

X	2.3	2.5	3.9	3.6	4.1	3.2	1.4	1.1
Y	8.6	8.1	7.6	7.8	7.4	6.6	5.2	6.1

6. (a) The following table shows how 10 students arranged in alphabetic order, were ranked according to their achievements in both the laboratory and lecture sections of physics course. Find the coefficient of rank correlation. [12] (CO3, PO1)

Laboratory	8	3	9	2	7	10	4	6	1	5
Lecture	9	4	10	1	8	7	3	6	2	5

(b) The mean breaking strength of a ropes manufactured by a company is 1300N and a standard deviation 100N. The company applied a new manufacturing process and claimed that the breaking strength has been increased. For this a sample of 35 ropes is tested and mean breaking strength is found to be 1325N. Can we support the claim at 0.05 significance level? [13] (CO3, PO1)