B.Sc. Eng.(CEE)/2nd Sem. May 20, 2024

ISLAMIC UNIVERSITY OF TECHNOLOGY (IUT)

ORGANISATION OF ISLAMIC COOPERATION (OIC)

DEPARTMENT OF NATURAL SCIENCES

Semester Final Examination — Summer Semester: 2022-2023

Course Number: Math 4253 — Full Marks: 150

Course Title: Vector Algebra, Vector Calculus, ODE — Time: 3 Hours

Answer all the **6 (Six)** questions. The symbols have their usual meanings. Marks of each question and the corresponding CO and PO are written in the brackets.

1. Solve the following differential equations:

(a) $(x^2 + y^2 + x)\,dx + xy\,dy = 0.$ **(8)** (CO1) (PO1)

(b) $xyp^2 + (3x^2 - 2y^2)p - 6xy = 0$, where $p = \frac{dy}{dx}$. **(9)** (CO1) (PO1)

(c) $(D^4 - 6D^3 + 13D^2 - 12D + 4)y = 0$, where $D \equiv \frac{d}{dx}$. **(8)** (CO1) (PO1)

2. (a) Solve: $(x^2D^2 - 2xD + 2)y = (\ln x)^2 - \ln x^2$. **(12)** (CO1) (PO1)

(b) In an L-R-C circuit an inductance of 0.05 henry, a resistor of 20 ohms and a capacitor of 100 microfarads have been connected through battery of e.m.f. $E = 100\cos 200t$ volts. At $t = 0$ the charge on the capacitor and current in a circuit are zero. Find the charge and current at any time $t > 0$. **(13)** (CO2) (PO2)

3. (a) Solve: $(D^2 + 3D + 2)y = x\sin 2x$. **(12)** (CO1) (PO1)

(b) If $(D-a)(D-b)y = Q$ then prove that $y = e^{ax}\int e^{(b-a)x}\int Qe^{-bx}(dx)^2$ and hence solve $(D^2 - 9D + 18)y = e^{e^{-3x}}$. **(13)** (CO2) (PO2)

4. (a) Discuss about curvature and torsion. Express curvature κ and torsion τ in terms of $\overline{r}$ and $\overline{s}$. **(8)** (CO3) (PO2)

(b) Find the unit tangent vector and unit normal vector at $t = 2$ on the curve $x = t^2 - 1, y = 4t - 3, z = 2t^2 - 6t$, where t is any variable. **(8)** (CO3) (PO2)

(c) Identify whether $u = x + y + z, v = x^2 + y^2 + z^2$ and $w = xy + yz + zx$ are functionally related or not. **(9)** (CO3) (PO2)

5.(a)	Find the maximum value of the directional derivative of $f(x,y,z) = x^2 - y^2 + z^2$ at the point (1,3,2). Find also the direction in which it occurs.	(5)	(CO3) (PO2)
(b)	If $\bar{r} = x\hat{\imath} + y\hat{\jmath} + z\hat{k}$, where $\lvert\bar{r}\rvert = r$, then find $\nabla^2 r^m$.	(8)	(CO1) (PO1)
(c)	A fluid motion is given by $\bar{v} = (y\sin z - \sin x)\hat{\imath} + (x\sin z + 2yz)\hat{\jmath} + (xy\cos z + y^2)\hat{k}$. Is the motion irrotational? If so, find the velocity potential.	(12)	(CO3) (PO2)
6.(a)	If $\bar{F} = 2x^2\hat{\imath} - xz\hat{\jmath} + y^2z\hat{k}$, evaluate $\iiint_V \bar{F}\,dV$, where V is the region bounded by the surfaces $x = 1, y = 0, y = 6, z = x^2, z = 4$.	(11)	(CO3) (PO2)
(b)	State Stoke's theorem. Apply Stoke's theorem to evaluate $\int_C (xydx + xy^2dy)$ taken round the positively oriented square C with vertices $(1,0), (-1,0), (0,1)$ and $(0,-1)$.	(14)	(CO3) (PO2)