BBA in TM, (2nd Sem.) May 14, 2024

ISLAMIC UNIVERSITY OF TECHNOLOGY (IUT)

ORGANISATION OF ISLAMIC COOPERATION (OIC)

DEPARTMENT OF NATURAL SCIENCES

Semester Final Examination | Summer Semester: A. Y. 2022-2023

Course no: Math-4261 | Time: 3 Hours

Course title: Mathematics II | Full Marks: 150

There are **6 (Six)** questions. Answer all of them. Programmable calculators are not allowed. Do not write anything on this question paper. Marks of each question and corresponding CO and PO are written in the right margin. The Symbols have their usual meaning.

			Marks	CO	PO
1.	**a)**	State the necessary and sufficient conditions for a function to be continuous and differentiable at a point.	(5)	1	1
	b)	A manufacturer of film developer finds that due to volume discounting the cost of producing x tons of the product is given (in terms of thousands of dollars) by $$C(x)A = \begin{cases} 3x+5;\ if\ 0 \le x \le 4 \\ 2x+6;\ if\ 4 < x \le 8 \\ \frac{1}{2}x+8;\ if\ 8 < x \le 16 \end{cases}$$ Discuss the continuity and differentiability of the cost function at x = 4 and 8 using the conditions.	(10)	2	1
	c)	If $\ln y = \tan^{-1} x$ then applying Leibnitz's theorem find $$(1+x^2)y_{n+2} + (2nx + 2x - 1)y_{n+1} + n(n+1)y_n.$$	(10)	3	2
2.	**a)**	If $f(x) = x^3 - 3x^2 + 1$ then use the first and second derivatives of f to determine the intervals on which f is increasing,decreasing, concave up, and concave down. Also locate all the inflection points.	(9)	2	1
	b)	If $u = tan^{-1}\frac{x^3+y^3}{x-y}$ then using Euler's theorem show that $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = sin2u$.	(8)	3	2
	c)	Find the Taylor Series for $f(x) = x^3 - 10x^2 + 6$ about x = 3.	(8)	2	1
3.	**a)**	A study prepared by the marketing department of the Universal Instruments Company forecasts that, after its new line of Galaxy Home Computers is introduced into the market, sales will grow at the rate of $$2000 - 1500e^{-0.05t} \qquad (0 \le t \le 60)$$ units per month. Use integration to find an expression that gives the total number of computers that will sell t months after they become available in the market.	(7)	3	2

How many computers will the company sell in the first year they are on the market?

b) Using the fundamental theorem of calculus find the area of the region under the graph of $y = x^2 + 1$ from $x = -1$ to $x = 2$. **(8)** 3 2

c) Clark County, Nevada, (dominated by Las Vegas) is the fastest growing metropolitan area in the United States. From 1970 through 2000, the population was growing at a rate of **(10)** 2 2

$R(t) = 133{,}680t^2 - 178{,}788t + 234{,}633,\ (0 \le t \le 3)$ people per decade, where t = 0 corresponds to the beginning of 1970. Find the net change in population over the decade from 1980 to 1990

4. a) Use the method of substitution to find value of the following: **(7)** 2 2

(i) $\int_0^4 x\sqrt{9 + x^2}\,dx$; (ii) $\int_0^1 \frac{x^2}{x^3+1}\,dx$

b) The interest rates changed by Madison Finance on auto loans for used cars over a certain 6-month period in 2008 are approximated by the function **(9)** 2 2

$$r(t) = -\frac{1}{12}t^3 + \frac{7}{8}t^2 - 3t + 12 \quad (0 \le t \le 6)$$

where t is measured in months and r(t) is the annual percentage rate.

Calculate the average rate on auto loans extended by Madison over the 6-month period.

c) The demand function for a certain make of Duranta bicycle is given by $p = D(x) = -0.0001x^2 + 25$, where p is the unit price in dollars and x is the quantity demanded in units of a thousand. The supply function for these bicycles is given by $p = S(x) = 0.00006x^2 + 0.002x + 10$, where p stands for the price in dollars and x stands for the number of bicycles that the supplier will want to sell. Determine the consumers' surplus and the producers' surplus if the market price of a bicycle is set at the equilibrium price. **(9)** 3 2

5. a) Write down the algorithm to solve $f(x) = 0$ by using bisection method. **(5)** 1 1

b) Find a real root of the equation $x^3 + x^2 - 1 = 0$ using bisection method. **(8)** 2 2

c) The following table gives the population of a town during the last six censuses. Estimate the increase in the population during the period from 1946 to 1948 using Newton's formula for forward interpolation: **(12)** 3 2

Year:	1911	1921	1931	1941	1951	1961
Population: (in thousands)	12	15	20	27	39	52

6. a) Write down the fourth order Runge-Kutta method to solve initial value problem. (5) 1 1

b) Apply the fourth order Runge-Kutta method to solve $\frac{dy}{dx} = x^2 + y^2$, $y(0) = 1$. (12) 4 2

Determine approximations to y (0.1) and y (0.2) correct to four decimal places, taking step size h = 0.1.

c) Using Newton-Raphson method find the smallest positive root of (8) 3 2

$x^3 - 5x + 3 = 0$.