B. Sc. Eng. (ME/IPE, 4th Sem.)/ B.Sc.TE 2Y (2nd Sem.) May 22, 2024

ISLAMIC UNIVERSITY OF TECHNOLOGY (IUT)

ORGANISATION OF ISLAMIC COOPERATION (OIC)

DEPARTMENT OF NATURAL SCIENCES

Final Semester Examination — Summer Semester: 2022-2023

Course No.: Math 4411 — Full Marks: 150

Course Title: Linear Algebra and Solid Geometry — Time: 3 Hours

There are **6 (Six)** questions. Answer all **6 (Six)** questions. Programmable calculators are not allowed. Do not write on this question paper. The figures in the right margin indicate full marks and corresponding CO and PO in the brackets. Symbols convey their usual meanings.

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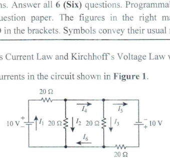
1. (a) Define Kirchhoff's Current Law and Kirchhoff's Voltage Law with examples. **(5)** (CO1) (PO1)

   (b) Find the various currents in the circuit shown in **Figure 1**. **(10)** (CO1) (PO2)



Figuer 1: Figure for Q1(b).

   (c) Discuss the conditions that must be satisfied for a set to qualify as a vector space? **(10)** (CO1) (PO2)

2. (a) Define linear dependence and linear independence in the context of vector spaces with examples. **(5)** (CO1) (PO1)

   (b) Find the real values of λ for which the following vectors form a linearly dependent set in R^3. **(10)** (CO1) (PO2)

$$\mathbf{v}_1 = \left(\lambda, -\frac{1}{2}, -\frac{1}{2}\right), \quad \mathbf{v}_2 = \left(-\frac{1}{2}, \lambda, -\frac{1}{2}\right), \quad \mathbf{v}_3 = \left(-\frac{1}{2}, -\frac{1}{2}, \lambda\right).$$

   (c) Determine whether the vectors $\mathbf{v}_1 = (1,1,2), \mathbf{v}_2 = (1,0,1)$, and $\mathbf{v}_3 = (2,1,3)$ span the vector space R^3. **(10)** (CO1) (PO2)

3. (a) Prove that the set of 2×2 matrices forms a vector space. **(5)** (CO1) (PO1)

   (b) Determine the dimension and a basis for the solution space of the system: **(10)** (CO1) (PO2)

$$x_1 - 3x_2 + x_3 = 0$$
$$2x_1 - 6x_2 + 2x_3 = 0$$
$$3x_1 - 9x_2 + 3x_3 = 0.$$

(c) Consider the basis $S = \{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3\}$ for R^3, where **(10)** (CO1) (PO2)

$$\mathbf{v}_1 = (1,1,1), \quad \mathbf{v}_2 = (1,1,0), \quad \mathbf{v}_3 = (1,0,0)$$

Let $T: R^3 \to R_2$ be the linear transformation for which

$$T(\mathbf{v}_1) = (1,0), \quad T(\mathbf{v}_2) = (2,-1), \quad T(\mathbf{v}_3) = (4,3)$$

Find a formula for $T(x_1, x_2, x_3)$, and then use that formula to compute $T(2,-3,5)$.

4. (a) Let $\mathbf{u} = (u_1, u_2)$ and $\mathbf{v} = (v_1, v_2)$ be vectors in R^2. Verify that the weighted Euclidean inner product **(5)** (CO2) (PO1)

$$\langle \mathbf{u}, \mathbf{v} \rangle = 3u_1v_1 + 2u_2v_2$$

satisfies the four inner product axioms.

(b) Assume that the vector space R^3 has the Euclidean inner product. Apply the Gram-Schmidt process to transform the basis vectors **(10)** (CO2) (PO2)

$$\mathbf{u}_1 = (1,1,1), \quad \mathbf{u}_2 = (0,1,1), \quad \mathbf{u}_3 = (0,0,1)$$

into an orthogonal basis $\{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3\}$, and then normalize the orthogonal basis vectors to obtain an orthonormal basis $\{\mathbf{q}_1, \mathbf{q}_2, \mathbf{q}_3\}$.

(c) Find bases for the eigenspaces of the matrix **(10)** (CO2) (PO2)

$$\begin{bmatrix} 0 & 0 & -2 \\ 1 & 2 & 1 \\ 1 & 0 & 3 \end{bmatrix}.$$

5. (a) Find the equation of the line perpendicular to both the lines $\frac{x-1}{1} = \frac{y-1}{2} = \frac{z+2}{3}$ $\frac{x+2}{2} = \frac{y-5}{-1} = \frac{z+3}{2}$; and passing through their intersection. **(8)** (CO3) (PO2)

(b) Find the length and equation of the shortest distance between lines $\frac{x-1}{2} = \frac{y-2}{3} = \frac{z-4}{4}$ and $\frac{x-2}{3} = \frac{y-4}{4} = \frac{z-5}{5}$. **(8)** (CO3) (PO2)

(c) Find the equation of the sphere passing through the points $(4,-1,2)$, $(0,-2,3)$, $(1,-5,-1)$ and $(2,0,1)$ **(9)** (CO3) (PO2)

6. (a) Find the equation of the paraboloid whose center is at the origin symmetric about the x-axis and passing through points $(1,2,2)$ and $(2,6,8)$. **(5)** (CO3) (PO2)

(b) Find the equation of the sphere that passing through the circle **(10)** (CO3) (PO2)

$x^2 + y^2 + z^2 - 2x + 3y - 4z + 6 = 0, 3x - 4y + 5z = 25$ cuts the shere

$x^2 + y^2 + z^2 + 2x + 4y - 6z + 11 = 0$ orthogolally.

(c) Tangent planes are drawn to the ellisoid $\frac{x^2}{a^2} + \frac{x^2}{a^2} + \frac{x^2}{a^2} = 1$ though $(\propto, \beta, \gamma)$, prove that perpendicular to them from the origin generates the surface **(10)** (CO3) (PO2)

$$(\alpha x + \beta y + \gamma z)^2 = a^2x^2 + b^2y^2 + c^2z^2.$$