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B. Sc. Engg. (CEE) 8th Sem. 14 May 2024 (Group A)

# ISLAMIC UNIVERSITY OF TECHNOLOGY (IUT)

ORGANISATION OF ISLAMIC COOPERATION (OIC)

**DEPARTMENT OF CIVIL AND ENVIRONMENTAL ENGINEERING**

**Final Examination** | **Summer Semester: 2022-2023**

**Course No.: CEE 4815** | **Full Marks: 150**

**Course Title: Introduction to Finite Element Method** | **Time: 3.0 Hours**

**There are 6 (Six) questions. Answer 5 questions. Questions 4, 5, and 6 are compulsory. Answer 2 (Two) questions from questions 1 to 3. The Symbols have their usual meaning. The figures in the right margin indicate full marks.**

1(a). Briefly explain the steps of the finite element method. (10) (CO1) (PO1)

1(b). Explain isoparametric element concept in finite element method. (5) (CO1) (PO1)

1(c). Derive the shape functions of 3-nodal truss element for the local coordinate system. (10) (CO1) (PO1)

2(a). Derive the shape functions and Jacobian matrix of a 4-nodal square element in the two-dimensional condition for the local coordinate system. (12) (CO1) (PO1)

2(b). Answer the following question regarding a quadrilateral element having coordinates of (1,1), (4, 1), (5, 2), and (2,4) - (13) (CO1) (PO1)

(i) Determine the coordinates of a point in the global coordinate system corresponding to the local coordinate (0.6, 0.7).

(ii) Determine Jacobian matrix

3. Answer the following questions for the finite element mesh shown in **Fig.1**. (25) (CO1) (PO1)

(i) Calculate the band width of the mesh.

(ii) Form $[K]_G \{\Delta\delta\}_{nG} = \{\Delta F\}_G$ considering 2 degrees of freedom of each node ($u$, $v$).

(iii) Include the boundary conditions in the global finite element equation where the nodal displacements at node 3 are ($u$ = 0.15m, $v$ = -0.25m).

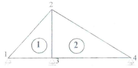



**Fig.1**

Stiffness matrix of each triangular element is –

$$[K^e] = \begin{bmatrix} K^e_{11} & K^e_{12} & K^e_{13} & K^e_{14} & K^e_{15} & K^e_{16} \\ K^e_{21} & K^e_{22} & K^e_{23} & K^e_{24} & K^e_{25} & K^e_{26} \\ K^e_{31} & K^e_{32} & K^e_{33} & K^e_{34} & K^e_{35} & K^e_{36} \\ K^e_{41} & K^e_{42} & K^e_{43} & K^e_{44} & K^e_{45} & K^e_{46} \\ K^e_{51} & K^e_{52} & K^e_{53} & K^e_{54} & K^e_{55} & K^e_{56} \\ K^e_{61} & K^e_{62} & K^e_{63} & K^e_{64} & K^e_{65} & K^e_{66} \end{bmatrix}$$

4(a). Derive the shape function for a 4-nodal interface element. (10) (CO1) (PO1)

4(b). Calculate the relative displacements of a interface element at the center (x = 0), left end (x = -L/2), and right end (x = L/2) for the mesh shown in **Fig.2**. The length of the interface element is 4.0m. Use the nodal displacement vector of the mesh – (15) (CO2) (PO2)

$$\{\delta^e\}^T = \{0.005 \quad -0.001 \quad 0.015 \quad -0.002 \quad 0.025 \quad 0.002 \quad 0.010 \quad 0.001\}$$

$$\{w\} = \frac{1}{2}[N]\{\delta^e\}$$


**Fig.2**

5. Answer the following questions considering plane strain condition for a triangular element shown in **Fig.3**. (55) (CO2) (PO2)

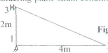



**Fig. 3**

(i) Calculate the Jacobian Matrix. The shape functions of the triangular element in local coordinate system are-

$$N_1(\xi,\eta) = 1-\xi-\eta,\; N_2(\xi,\eta) = \xi, \text{ and } N_3(\xi,\eta) = \eta$$

(ii) Calculate strain-displacement matrix (**B** Matrix).

(iii) Calculate elastic stress-strain matrix (**D** Matrix) for plane strain condition using Lame's constants ($\lambda$=40 MPa and $\mu$ = 40 MPa).

(iv) Calculate stiffness matrix (**K** Matrix).

(v) Considering displacements of node 2 (20mm, -40mm), and node 3 (0,-10mm), form nodal displacement vector $\{d\delta\}$.

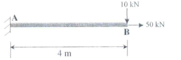
(vi) Calculate strain increment vector of the element, $\{\Delta\varepsilon\} = [B]\{\Delta\delta\}$.

(vii) Calculate stress increment vector of the element, $\{\Delta\sigma\} = [D]\{\Delta\varepsilon\}$.

(viii) Calculate nodal forces of the element $\{\Delta F\}$.

6(a). Apply boundary conditions of the cantilever beam shown in **Fig.4** in the finite element equation **[K]{Δδ}= {ΔF}**. Also, calculate horizontal displacement ($u$), vertical deflection ($v$), and rotation ($\theta$) at point B for the beam. Use, axial stiffness $EA$ = 10,000 kN, and flexural stiffness $EI$ = 1,000 kN-m$^2$ of the beam element. (15) (CO2) (PO2)

**Fig.4**




Stiffness matrix of a beam element is-

$$[k]=\begin{bmatrix} \frac{EA}{l} & 0 & 0 & -\frac{EA}{l} & 0 & 0 \\ 0 & 12\frac{EI}{l^3} & 6\frac{EI}{l^2} & 0 & -12\frac{EI}{l^3} & 6\frac{EI}{l^2} \\ 0 & 6\frac{EI}{l^2} & 4\frac{EI}{l} & 0 & -6\frac{EI}{l^2} & 2\frac{EI}{l} \\ -\frac{EA}{l} & 0 & 0 & \frac{EA}{l} & 0 & 0 \\ 0 & -12\frac{EI}{l^3} & -6\frac{EI}{l^2} & 0 & 12\frac{EI}{l^3} & -6\frac{EI}{l^2} \\ 0 & 6\frac{EI}{l^2} & 2\frac{EI}{l} & 0 & -6\frac{EI}{l^2} & 4\frac{EI}{l} \end{bmatrix}$$

6(b). A circular truss member with the length of 10m and diamerer of 0.20m is subjected to a dynamic loading. Calculate the consistent mass matrix of the truss member, where the density of the truss material is 5000 kg/$m^3$. (5) (CO2) (PO2)