B.Sc. Engg. CSE 4th Semester 27 May 2024

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ORGANISATION OF ISLAMIC COOPERATION (OIC)

Department of Computer Science and Engineering (CSE)

SEMESTER FINAL EXAMINATION SUMMER SEMESTER, 2022-2023

DURATION: 3 HOURS FULL MARKS: 150

Math 4441: Probability and Statistics

Programmable calculators are not allowed. Do not write anything on the question paper.
Answer **all 7 (seven)** questions. Figures in the right margin indicate full marks of questions with corresponding COs and POs in parentheses. Assume any missing values.

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1. a) There are 100 questions in a test. Suppose that, for all $s > 0$ and $t > 0$, the event that it takes t minutes to answer one question is independent of the event that it takes s minutes to answer another one. If the time that it takes to answer a question is exponential with mean 1/2, find the distribution, the average time, and the standard deviation of the time it takes to do the entire test. 10 (CO1) (PO1)

   b) Suppose that b is a random number from the interval $(-3, 3)$. Determine the probability that the quadratic equation $x^2 + bx + 1 = 0$ has at least one real root. 10 (CO1) (PO1)

   c) The scores on an achievement test given to 100,000 students are normally distributed with mean 500 and standard deviation 100. What should the score of a student be to place him among the top 10% of all students? 10 (CO1) (PO1)

2. A coin, having probability p of landing heads, is continually flipped until at least one head and one tail have been flipped. (CO2) (PO2)

   a) Find the expected number of flips needed. 10

   b) Find the expected number of flips that land on heads. 5

   c) Find the expected number of flips that land on tails. 5

   d) Repeat Question 2.a in the case where flipping is continued until a total of at least two heads and one tail have been flipped. 5

3. a) Let X and Y be independent exponential random variables, each with parameter λ. Find the distribution function of $X + Y$. 10 (CO2) (PO2)

   b) Suppose that the time it takes for a novice secretary to type a document is exponential with mean 1 hour. If at the beginning of a certain eight-hour working day, the secretary receives 12 documents to type, find the probability that she will finish them all by the end of the day. 10 (CO1) (PO1)

4. Let $\mathbf{X}$ is a 3-dimensional Gaussian random vector with expected value $\mu_{\mathbf{X}} = [4 \quad 8 \quad 6]^{\top}$ and covariance matrix (CO1) (PO1)

$$\mathbf{C_X} = \begin{bmatrix} 4 & -2 & 1 \\ -2 & 4 & -2 \\ 1 & -2 & 4 \end{bmatrix}$$

   a) Calculate the correlation matrix, $\mathbf{R_X}$. 4

   b) Calculate the joint PDF of the first two components of $\mathbf{X}$, $f_{X_1,X_2}(x_1, x_2)$. 7

c) Calculate the covariance matrix of $\mathbf{Y} = \mathbf{A}\mathbf{X} + \mathbf{b}$, where 4

$$\mathbf{A} = \begin{bmatrix} 1 & 1/2 & 2/3 \\ 1 & -1/2 & 2/3 \end{bmatrix} \text{ and } \mathbf{b} = [-4 \quad -4]^\mathsf{T}.$$

5. a) Suppose X is a discrete random variable with the following probability mass function: 10 (CO3) (PO4)

x	0	1	2	3
$P_X(x)$	$2\theta/3$	$\theta/3$	$2(1-\theta)/3$	$(1-\theta)/3$

where $0 \le \theta \le 1$ is a parameter. The following 10 independent observations were taken from such a distribution: $(3, 0, 2, 1, 3, 2, 1, 0, 2, 1)$.

Find the maximum likelihood estimate of the parameter θ.

b) Suppose $X_1, X_2, \dots, X_n$ are i.i.d. random variables with the following probability density function: 10 (CO3) (PO4)

$$f_X(x|\sigma) = \begin{cases} \frac{1}{2\sigma}\exp\left(-\frac{|x|}{\sigma}\right), & \text{for } -\infty \le x \le +\infty; \\ 0, & \text{otherwise.} \end{cases}$$

Find the maximum likelihood estimate of σ.

6. a) Apple Tree Supermarket is considering opening a new store at a certain location, but wants to know if average weekly sales will reach \$250,000. Apple Tree estimates weekly gross sales at nearby stores by sending field workers to collect observations. The field workers collect 40 samples and arrive at a sample mean of \$263,590. Suppose the standard deviation of the sales is known to be \$42,000. Find a 90% confidence interval for the population mean μ. 8 (CO3) (PO4)

b) Suppose we monitor 30 computer programmers with and without music. Suppose the sample average number of lines of code produced without music is 195 and with music that is 185. Suppose the population variances for both cases are known and have respective values of 580 and 444 for programmers without and with music. Find the 95% confidence interval for the difference of average number of lines of code produced without and with music. 10 (CO3) (PO4)

7. a) Bottles of mango juice nominally contain 1000 millilitres. After the introduction of a new method of filling the bottles, there is a suspicion that the mean volume of mango juice in a bottle has changed. 10 (CO3) (PO4)

In order to investigate the suspicion, a random sample of 12 bottles of mango juice is taken and the volume of mango juice in each is measured.

The volumes, in millilitres, of mango juice in these bottles are found to be:

996 1006 1009 999 1007 1003
998 1010 997 996 1008 1007

Assuming that the volume of mango juice in a bottle is normally distributed, investigate, at the level of 5% significance, whether the mean volume of mango juice in a bottle differs from 1000 millilitres.

b) An independent testing agency was hired to study whether or not the work output is different for construction workers employed by the state and receiving prevailing wages versus construction workers in the private sector who are paid rates determined by the free market. A sample of 10 state workers reveals an average output of 69.7 parts per hour with a sample standard deviation of 18 parts per hour. A sample of 10 private sector workers reveals an average output of 74.3 parts per hour with a sample standard deviation of 16 parts per hour. In developing your answer, you may assume that the unknown variances are equal. Construct a hypothesis test at the 0.10 level of significance indicating whether there is any evidence of a difference in the productivity level of the state and private sector workers. Also, find the p-value of the test. 12 (CO3) (PO4)

Appendix - A

PMF/PDF, expected values and variance of known Random Variables

Families of Distribution	PMF or PDF	Expectation	Variance
Bernoulli $X \sim Ber(p)$	$P_X(x) = \begin{cases} p^x(1-p)^{1-x}, & x = 0, 1 \\ 0, & \text{otherwise.} \end{cases}$	p	$p(1-p)$
Geometric $X \sim Geom(p)$	$P_X(x) = \begin{cases} p(1-p)^{x-1}, & x = 1, 2, \dots \\ 0, & \text{otherwise.} \end{cases}$	$\frac{1}{p}$	$\frac{(1-p)}{p^2}$
Binomial $X \sim Bin(n, p)$	$P_X(x) = \begin{cases} \binom{n}{x}p^x(1-p)^{n-x}, & x = 0, 1, \dots, n \\ 0, & \text{otherwise.} \end{cases}$	np	$np(1-p)$
Pascal/-ve Binomial $X \sim pascal(k, p)$	$P_X(x) = \begin{cases} \binom{x-1}{k-1}p^k(1-p)^{x-k}, & x = k, k+1, \dots \\ 0, & \text{otherwise.} \end{cases}$	$\frac{k}{p}$	$\frac{k(1-p)}{p^2}$
Poisson $X \sim Poisson(\lambda)$	$P_X(x) = \begin{cases} \frac{(\lambda T)^x e^{-(\lambda T)}}{x!}, & x \geq 0 \\ 0, & \text{otherwise.} \end{cases}$	λT	λT
Hyper Geometric $X \sim HGem(r, g, n)$	$P_X(x) = \begin{cases} \frac{\binom{r}{x}\binom{g}{n-x}}{\binom{r+g}{n}}, & x = 0, 1, \dots, \min(r, n) \\ 0, & \text{otherwise.} \end{cases}$	$\frac{rn}{r+g}$	$\frac{nrg}{(r+g)^2}(1-\frac{n-1}{r+g-1})$
Uniform (discrete) $X \sim unif(a, b)$	$P_X(x) = \begin{cases} \frac{1}{b-a+1}, & x = a, a+1, \dots, b \\ 0, & \text{otherwise.} \end{cases}$	$\frac{a+b}{2}$	$\frac{(b-a)(b-a+2)}{12}$
Exponential $X \sim exp(\lambda)$	$f_X(x) = \begin{cases} \lambda e^{-\lambda x}, & x \geq 0 \\ 0, & \text{otherwise.} \end{cases}$	$\frac{1}{\lambda}$	$\frac{1}{\lambda^2}$
Gaussian $X \sim N(\mu, \sigma^2)$	$f_X(x) = \begin{cases} \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{(x-\mu)^2}{2\sigma}}, & -\infty < x < +\infty \\ 0, & \text{otherwise.} \end{cases}$	μ	σ^2
Uniform (Continuous) $X \sim unif(a, b)$	$f_X(x) = \begin{cases} \frac{1}{b-a}, & a \leq x \leq b \\ 0, & \text{otherwise.} \end{cases}$	$\frac{a+b}{2}$	$\frac{(b-a)^2}{12}$
Gamma $X \sim gam(r, \lambda)$	$f_X(x) = \begin{cases} \frac{\lambda e^{-\lambda x}(\lambda x)^{r-1}}{\Gamma(r)}, & x \geq 0 \\ 0, & \text{otherwise.} \end{cases}$	$\frac{r}{\lambda}$	$\frac{r}{\lambda^2}$
Multinomial	$P_{X_1,\dots,X_r}(x_1, \dots, x_r) = \begin{cases} \binom{n}{x_1 \dots x_r} p_1^{x_1} \dots p_r^{x_r}, & x_1 = 0, \dots, n, 0 \leq i \leq r, \sum_{i=0}^{r} x_i = n \\ 0, & \text{otherwise.} \end{cases}$		
Multivariate Hyper geometric	$P_X(x) = \begin{cases} \frac{\binom{n_1}{x_1}\binom{n_2}{x_2}\dots\binom{n_r}{x_r}}{\binom{n_1+\dots+n_r}{x_1+\dots+x_r}}, & x = 0, 1, \dots, \min(r, n) \\ 0, & \text{otherwise.} \end{cases}$		

Appendix - B

General Formulas

Name	Formula
Conditional Probability	$P[A\|B] = \frac{P[AB]}{P[B]}, P[B] > 0$
Product Rule	$P[AB] = P[A]P[B\|A]$ $P[A_1 \dots A_n] = P[A_1]P[A_2\|A_1] \dots P[A_n\|A_1 \dots A_{n-1}]$
Conditional Product Rule	$P[AB\|C] = P[A\|C]P[B\|AC]$
Sum Rule	$P[A] = \sum_{i=1}^{n} P[A\|B_i]P[B_i]$
Conditional Sum Rule	$P[A\|C] = \sum_{i=1}^{n} P[A\|B_i, C]P[B_i\|C]$
Bayes' Theorem	$P[B_i\|A] = \frac{P[A\|B_i]P[B_i]}{\sum_{i=1}^{n} P[A\|B_i]P[B_i]}$
Conditional Bayes' Theorem	$P[B_i\|A, C] = \frac{P[A\|B_i,C]P[B_i\|C]}{\sum_{i=1}^{n} P[A\|B_i,C]P[B_i\|C]}$
Independence of Events	$P[AB] = P[A]P[B]$
Expected Value	$E[X] = \sum_{x \in S_X} xP_X(x)$ $E[X] = \int_{-\infty}^{+\infty} xf_X(x)dx$
Variance	$Var[X] = E[(X - \mu_X)^2] = E[X^2] - (\mu_X)^2$
Marginal PMF	$P_X(x) = \sum_{x \in S_X} P_{XY}(x, y)$ $f_X(x) = \int_{-\infty}^{+\infty} f_{XY}(x, y)dy$
Covariance	$Cov[X, Y] = E[(X - \mu_X)(Y - \mu_Y)] = E[XY] - \mu_X\mu_Y$
Correlation	$r_{XY} = E[XY]$
Correlation Coefficient	$\rho_{XY} = \frac{Cov[X,Y]}{\sqrt{Var[X]Var[Y]}}$
Conditional Distribution	$P_{XY}(x, y) = P_{X\|Y}(x\|y)P_Y(y)$ $P_X(x) = \sum_y P_{X\|Y}(x\|y)P_Y(y)$ $E[X] = \sum_y E[X\|Y = y]P_Y(y)$ $f_{XY}(x, y) = f_{X\|Y}(x\|y)f_Y(y)$ $f_X(x) = \int f_{X\|Y}(x\|y)f_Y(y)dy$ $E[X] = \int E[X\|Y = y]f_Y(y)dy$
Random Vector	$f_{\mathbf{Y(y)}} = \frac{1}{\|det(\mathbf{A}\|} f_{\mathbf{X}}(\mathbf{A}^{-1}(\mathbf{y} - \mathbf{b}))$ $E[\mathbf{X}] = [E[X_1] \quad E[X_2] \quad \dots \quad E[X_n]]^T$ $\mathbf{R_X} = E[\mathbf{XX}^T]$ $\mathbf{C_X} = E[(\mathbf{X} - \mu_{\mathbf{X}})(\mathbf{X} - \mu_{\mathbf{X}})^T]$ $\mathbf{C_X} = \mathbf{R_X} - \mu_{\mathbf{X}}\mu_{\mathbf{X}}^T$ If $\mathbf{Y} = \mathbf{AX} + \mathbf{b}$ $\mu_{\mathbf{Y}} = \mathbf{A}\mu_{\mathbf{X}} + \mathbf{b}$ $\mathbf{R_Y} = \mathbf{AR_XA}^T + (\mathbf{A}\mu_{\mathbf{X}})\mathbf{b}^T + \mathbf{b}(\mathbf{A}\mu_{\mathbf{X}})^T + \mathbf{bb}^T$ $\mathbf{C_Y} = \mathbf{AC_XA}^T$
Gaussian Random Vector	$f_{\mathbf{X}}(\boldsymbol{x}) = \frac{1}{(2\pi)^{n/2}[det(\mathbf{C_X})]^{1/2}} \exp\left(-\frac{1}{2}(\boldsymbol{x} - \mu_{\mathbf{X}})^{\mathrm{T}}\mathbf{C_X}^{-1}(\boldsymbol{x} - \mu_{\mathbf{X}})\right)$ $E[\mathbf{X}] = [E[X_1] \quad E[X_2] \quad \dots \quad E[X_n]]^T$ If $\mathbf{Y} = \mathbf{AX} + \mathbf{b}$ $\mu_{\mathbf{Y}} = \mathbf{A}\mu_{\mathbf{X}} + \mathbf{b}$ $\mathbf{C_Y} = \mathbf{AC_XA}^T$

Table 7.1 $100(1-\alpha)$ Percent Confidence Intervals

$X_1, \ldots, X_n \sim \mathcal{N}(\mu, \sigma^2)$ $\quad \overline{X} = \sum_{i=1}^{n} X_i/n, \qquad S = \sqrt{\sum_{i=1}^{n}(X_i - \overline{X})^2/(n-1)}$

Assumption	Parameter	Confidence Interval	Lower Interval	Upper Interval
σ^2 known	μ	$\overline{X} \pm z_{\alpha/2}\frac{\sigma}{\sqrt{n}}$	$\left(-\infty, \overline{X}+z_\alpha \frac{\sigma}{\sqrt{n}}\right)$	$\left(\overline{X}+z_\alpha \frac{\sigma}{\sqrt{n}}, \infty\right)$
σ^2 unknown	μ	$\overline{X} \pm t_{\alpha/2,\,n-1}\frac{S}{\sqrt{n}}$	$\left(-\infty, \overline{X}+t_{\alpha,\,n-1} \frac{S}{\sqrt{n}}\right)$	$\left(\overline{X}-t_{\alpha,\,n-1} \frac{S}{\sqrt{n}}, \infty\right)$
μ unknown	σ^2	$\left(\frac{(n-1)S^2}{\chi^2_{\alpha/2,\,n-1}}, \frac{(n-1)S^2}{\chi^2_{1-\alpha/2,\,n-1}}\right)$	$\left(0, \frac{(n-1)S^2}{\chi^2_{1-\alpha,\,n-1}}\right)$	$\left(\frac{(n-1)S^2}{\chi^2_{\alpha,\,n-1}}, \infty\right)$

Table 7.2 $100(1-\sigma)$ Percent Confidence Intervals for $\mu_1 - \mu_2$

$X_1, \ldots, X_n \sim \mathcal{N}(\mu_1, \sigma_1^2)$

$Y_1, \ldots, Y_m \sim \mathcal{N}(\mu_2, \sigma_2^2)$

$\overline{X} = \sum_{i=1}^{n} X_i/n, \quad S_1^2 = \sum_{i=1}^{n}(X_i - \overline{X})^2/(n-1)$

$\overline{Y} = \sum_{i=1}^{m} Y_i/n, \quad S_2^2 = \sum_{i=1}^{m}(Y_i - \overline{Y})^2/(m-1).$

Assumption	Confidence Interval
σ_1, σ_2 known	$\overline{X} - \overline{Y} \pm z_{\alpha/2}\sqrt{\sigma_1^2/n + \sigma_2^2/m}$
σ_1, σ_2 unknown but equal	$\overline{X} - \overline{Y} \pm t_{\alpha/2,\,n+m-2}\sqrt{\left(\frac{1}{n}+\frac{1}{m}\right)\frac{(n-1)S_1^2 + (m-1)S_2^2}{n+m-2}}$
Assumption	**Lower Confidence Interval**
σ_1, σ_2 known	$(-\infty, \overline{X} - \overline{Y} + z_{\alpha}\sqrt{\sigma_1^2/n + \sigma_2^2/m})$
σ_1, σ_2 unknown but equal	$\left(-\infty, \overline{X} - \overline{Y} + t_{\alpha,\,n+m-2}\sqrt{\left(\frac{1}{n}+\frac{1}{m}\right)\frac{(n-1)S_1^2 + (m-1)S_2^2}{n+m-2}}\right)$

Note: Upper confidence intervals for $\mu_1 - \mu_2$ are obtained from lower confidence intervals for $\mu_2 - \mu_1$.

Table 7.3 Approximate $100(1-\alpha)$ Percent Confidence Intervals for p

X Is a Binomial (n, p) Random Variable $\hat{p} = X/n$

Type of Interval	Confidence Interval
Two-sided	$\hat{p} \pm z_{\alpha/2}\sqrt{\hat{p}(1-\hat{p})/n}$
One-sided lower	$\left(-\infty, \hat{p}+z_\alpha\sqrt{\hat{p}(1-\hat{p})/n}\right)$
One-sided upper	$\left(\hat{p}-z_\alpha\sqrt{\hat{p}(1-\hat{p})/n}, \infty\right)$

Table 8.1 $X_1, \ldots, X_n$ Is a Sample from a (μ, σ^2) Population, σ^2 Is Known, $\overline{X} = \sum_{i=1}^{n} X_i/n$.

H_0	H_1	Test Statistic TS	Significance Level α Test	p-Value if $TS = t$
$\mu = \mu_0$	$\mu \neq \mu_0$	$\sqrt{n}(\overline{X} - \mu_0)/\sigma$	Reject if $\|TS\| > z_{\alpha/2}$	$2P\{Z \geq \|t\|\}$
$\mu \leq \mu_0$	$\mu > \mu_0$	$\sqrt{n}(\overline{X} - \mu_0)/\sigma$	Reject if $TS > z_\alpha$	$P\{Z \geq t\}$
$\mu \geq \mu_0$	$\mu < \mu_0$	$\sqrt{n}(\overline{X} - \mu_0)/\sigma$	Reject if $TS < -z_\alpha$	$P\{Z \leq t\}$

Z is a standard normal random variable.

Table 8.2 $X_1, \ldots, X_n$ Is a Sample from a (μ, σ^2) Population, σ^2 Is Unknown, $\overline{X} = \sum_{i=1}^{n} X_i/n S^2 = \sum_{i=1}^{n} (X_i - \overline{X})^2/(n-1)$.

H_0	H_1	Test Statistic TS	Significance Level α Test	p-Value if $TS = t$
$\mu = \mu_0$	$\mu \neq \mu_0$	$\sqrt{n}(\overline{X} - \mu_0)/S$	Reject if $\|TS\| > t_{\alpha/2,\, n-1}$	$2P\{T_{n-1} \geq \|t\|\}$
$\mu \leq \mu_0$	$\mu > \mu_0$	$\sqrt{n}(\overline{X} - \mu_0)/S$	Reject if $TS > t_{\alpha,\, n-1}$	$P\{T_{n-1} \geq t\}$
$\mu \geq \mu_0$	$\mu < \mu_0$	$\sqrt{n}(\overline{X} - \mu_0)/S$	Reject if $TS < -t_{\alpha,\, n-1}$	$P\{T_{n-1} \leq t\}$

T_{n-1} is a t-random variable with $n-1$ degrees of freedom: $P\{T_{n-1} > t_{\alpha,\, n-1}\} = \alpha$.

Table 8.4 $X_1, \ldots, X_n$ Is a Sample from a (μ_1, σ_1^2) Population; $Y_1, \ldots, Y_m$ Is a Sample from a (μ_2, σ_2^2) Population,
The Two Population Samples Are Independent to Test
$H_0 : \mu_1 = \mu_2$ versus $H_0 : \mu_1 \neq \mu_2$.

Assumption	Test Statistic TS	Significance Level α Test	p-Value if $TS = t$
σ_1, σ_2 known	$\frac{\overline{X} - \overline{Y}}{\sqrt{\sigma_1^2/n + \sigma_2^2/m}}$	Reject if $\|TS\| > z_{\alpha/2}$	$2P\{Z \geq \|t\|\}$
$\sigma_1 = \sigma_2$	$\frac{\overline{X} - \overline{Y}}{\sqrt{\frac{(n-1)S_1^2 + (m-1)S_2^2}{n+m-2}}\sqrt{1/n + 1/m}}$	Reject if $\|TS\| > t_{\alpha/2,\, n+m-2}$	$2P\{T_{n+m-2} \geq \|t\|\}$
n, m large	$\frac{\overline{X} - \overline{Y}}{\sqrt{S_1^2/n + S_2^2/m}}$	Reject if $\|TS\| > z_{\alpha/2}$	$2P\{Z \geq \|t\|\}$

$$\Phi(z) = P(Z \leq z) = \int_{-\infty}^{z} \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}u^2} \, du$$

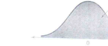

TABLE III Cumulative Standard Normal Distribution (*continued*)

z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.500000	0.503989	0.507978	0.511967	0.515953	0.519939	0.532922	0.527903	0.531881	0.535856
0.1	0.539828	0.543795	0.547758	0.551717	0.555760	0.559618	0.563559	0.567495	0.571424	0.575345
0.2	0.579260	0.583166	0.587064	0.590954	0.594835	0.598706	0.602568	0.606420	0.610261	0.614092
0.3	0.617911	0.621719	0.625516	0.629300	0.633072	0.636831	0.640576	0.644309	0.648027	0.651732
0.4	0.655422	0.659097	0.662757	0.666402	0.670031	0.673645	0.677242	0.680822	0.684386	0.687933
0.5	0.691462	0.694974	0.698468	0.701944	0.705401	0.708840	0.712260	0.715661	0.719043	0.722405
0.6	0.725747	0.729069	0.732371	0.735653	0.738914	0.742154	0.745373	0.748571	0.751748	0.754903
0.7	0.758036	0.761148	0.764238	0.767305	0.770350	0.773373	0.776373	0.779350	0.782305	0.785236
0.8	0.788145	0.791030	0.793892	0.796731	0.799546	0.802338	0.805106	0.807850	0.810570	0.813267
0.9	0.815940	0.818589	0.821214	0.823815	0.826391	0.828944	0.831472	0.833977	0.836457	0.838913
1.0	0.841345	0.843752	0.846136	0.848495	0.850830	0.853141	0.855428	0.857690	0.859929	0.862143
1.1	0.864334	0.866500	0.868643	0.870762	0.872857	0.874928	0.876976	0.878999	0.881000	0.882977
1.2	0.884930	0.886860	0.888767	0.890651	0.892512	0.894350	0.896165	0.897958	0.899727	0.901475
1.3	0.903199	0.904902	0.906582	0.908241	0.909877	0.911492	0.913085	0.914657	0.916207	0.917736
1.4	0.919243	0.920730	0.922196	0.923641	0.925066	0.926471	0.927855	0.929219	0.930563	0.931888
1.5	0.933193	0.934478	0.935744	0.936992	0.938220	0.939429	0.940620	0.941792	0.942947	0.944083
1.6	0.945201	0.946301	0.947384	0.948449	0.949497	0.950529	0.951543	0.952540	0.953521	0.954486
1.7	0.955435	0.956367	0.957284	0.958185	0.959071	0.959941	0.960796	0.961636	0.962462	0.963273
1.8	0.964070	0.964852	0.965621	0.966375	0.967116	0.967843	0.968557	0.969258	0.969946	0.970621
1.9	0.971283	0.971933	0.972571	0.973197	0.973810	0.974412	0.975002	0.975581	0.976148	0.976705
2.0	0.977250	0.977784	0.978308	0.978822	0.979325	0.979818	0.980301	0.980774	0.981237	0.981691
2.1	0.982136	0.982571	0.982997	0.983414	0.983823	0.984222	0.984614	0.984997	0.985371	0.985738
2.2	0.986097	0.986447	0.986791	0.987126	0.987455	0.987776	0.988089	0.988396	0.988696	0.988989
2.3	0.989276	0.989556	0.989830	0.990097	0.990358	0.990613	0.990863	0.991106	0.991344	0.991576
2.4	0.991802	0.992024	0.992240	0.992451	0.992656	0.992857	0.993053	0.993244	0.993431	0.993613
2.5	0.993790	0.993963	0.994132	0.994297	0.994457	0.994614	0.994766	0.994915	0.995060	0.995201
2.6	0.995339	0.995473	0.995604	0.995731	0.995855	0.995975	0.996093	0.996207	0.996319	0.996427
2.7	0.996533	0.996636	0.996736	0.996833	0.996928	0.997020	0.997110	0.997197	0.997282	0.997365
2.8	0.997445	0.997523	0.997599	0.997673	0.997744	0.997814	0.997882	0.997948	0.998012	0.998074
2.9	0.998134	0.998193	0.998250	0.998305	0.998359	0.998411	0.998462	0.998511	0.998559	0.998605
3.0	0.998650	0.998694	0.998736	0.998777	0.998817	0.998856	0.998893	0.998930	0.998965	0.998999
3.1	0.999032	0.999065	0.999096	0.999126	0.999155	0.999184	0.999211	0.999238	0.999264	0.999289
3.2	0.999313	0.999336	0.999359	0.999381	0.999402	0.999423	0.999443	0.999462	0.999481	0.999499
3.3	0.999517	0.999533	0.999550	0.999566	0.999581	0.999596	0.999610	0.999624	0.999638	0.999650
3.4	0.999663	0.999675	0.999687	0.999698	0.999709	0.999720	0.999730	0.999740	0.999749	0.999758
3.5	0.999767	0.999776	0.999784	0.999792	0.999800	0.999807	0.999815	0.999821	0.999828	0.999835
3.6	0.999841	0.999847	0.999853	0.999858	0.999864	0.999869	0.999874	0.999879	0.999883	0.999888
3.7	0.999892	0.999896	0.999900	0.999904	0.999908	0.999912	0.999915	0.999918	0.999922	0.999925
3.8	0.999928	0.999931	0.999933	0.999936	0.999938	0.999941	0.999943	0.999946	0.999948	0.999950
3.9	0.999952	0.999954	0.999956	0.999958	0.999959	0.999961	0.999963	0.999964	0.999966	0.999967

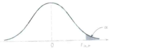

TABLE V Percentage Points $t_{\alpha,\nu}$ of the t Distribution

ν \ α	.40	.25	.10	.05	.025	.01	.005	.0025	.001	.0005
1	.325	1.000	3.078	6.314	12.706	31.821	63.657	127.32	318.31	636.62
2	.289	.816	1.886	2.920	4.303	6.965	9.925	14.089	23.326	31.598
3	.277	.765	1.638	2.353	3.182	4.541	5.841	7.453	10.213	12.924
4	.271	.741	1.533	2.132	2.776	3.747	4.604	5.598	7.173	8.610
5	.267	.727	1.476	2.015	2.571	3.365	4.032	4.773	5.893	6.869
6	.265	.718	1.440	1.943	2.447	3.143	3.707	4.317	5.208	5.959
7	.263	.711	1.415	1.895	2.365	2.998	3.499	4.029	4.785	5.408
8	.262	.706	1.397	1.860	2.306	2.896	3.355	3.833	4.501	5.041
9	.261	.703	1.383	1.833	2.262	2.821	3.250	3.690	4.297	4.781
10	.260	.700	1.372	1.812	2.228	2.764	3.169	3.581	4.144	4.587
11	.260	.697	1.363	1.796	2.201	2.718	3.106	3.497	4.025	4.437
12	.259	.695	1.356	1.782	2.179	2.681	3.055	3.428	3.930	4.318
13	.259	.694	1.350	1.771	2.160	2.650	3.012	3.372	3.852	4.221
14	.258	.692	1.345	1.761	2.145	2.624	2.977	3.326	3.787	4.140
15	.258	.691	1.341	1.753	2.131	2.602	2.947	3.286	3.733	4.073
16	.258	.690	1.337	1.746	2.120	2.583	2.921	3.252	3.686	4.015
17	.257	.689	1.333	1.740	2.110	2.567	2.898	3.222	3.646	3.965
18	.257	.688	1.330	1.734	2.101	2.552	2.878	3.197	3.610	3.922
19	.257	.688	1.328	1.729	2.093	2.539	2.861	3.174	3.579	3.883
20	.257	.687	1.325	1.725	2.086	2.528	2.845	3.153	3.552	3.850
21	.257	.686	1.323	1.721	2.080	2.518	2.831	3.135	3.527	3.819
22	.256	.686	1.321	1.717	2.074	2.508	2.819	3.119	3.505	3.792
23	.256	.685	1.319	1.714	2.069	2.500	2.807	3.104	3.485	3.767
24	.256	.685	1.318	1.711	2.064	2.492	2.797	3.091	3.467	3.745
25	.256	.684	1.316	1.708	2.060	2.485	2.787	3.078	3.450	3.725
26	.256	.684	1.315	1.706	2.056	2.479	2.779	3.067	3.435	3.707
27	.256	.684	1.314	1.703	2.052	2.473	2.771	3.057	3.421	3.690
28	.256	.683	1.313	1.701	2.048	2.467	2.763	3.047	3.408	3.674
29	.256	.683	1.311	1.699	2.045	2.462	2.756	3.038	3.396	3.659
30	.256	.683	1.310	1.697	2.042	2.457	2.750	3.030	3.385	3.646
40	.255	.681	1.303	1.684	2.021	2.423	2.704	2.971	3.307	3.551
60	.254	.679	1.296	1.671	2.000	2.390	2.660	2.915	3.232	3.460
120	.254	.677	1.289	1.658	1.980	2.358	2.617	2.860	3.160	3.373
∞	.253	.674	1.282	1.645	1.960	2.326	2.576	2.807	3.090	3.291