Hybrid Grey Wolf Algorithm Development and Analysis for Effective Multi-Objective Optimization

2024

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A Thesis by

Md Fahim Tanvir Azad (190012128)
Rifat bin Kawsar (190012131)

A Thesis Submitted in Partial Fulfillment of the Requirement for the Degree of Bachelor of Science in Industrial and Production Engineering

Department of Mechanical and Production Engineering (MPE)
Islamic University of Technology (IUT)

June, 2024

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<u>CERTIFICATE OF RESEARCH</u>

This thesis titled "HYBRID GREY WOLF ALGORITHM DEVELOPMENT AND ANALYSIS FOR EFFECTIVE MULTI- OBJECTIVE OPTIMIZATION" submitted by MD. FAHIM TANVIR AZAD (190012128) and RIFAT BIN KAWSAR (190012131) has been accepted as satisfactory in partial fulfillment of the requirement for the Degree of Bachelor of Science in Industrial and Production Engineering.

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Candidate's Declaration

This is to certify that the work presented in this thesis, titled**, "Hybrid Grey Wolf Algorithm Development and Analysis for Effective Multi Objective Optimization"**, is the outcome of the investigation and research carried out by me under the supervision of **Prof. Dr. Mohammad Ahsan Habib, PhD**

It is also declared that neither this thesis nor any part of it has been submitted elsewhere for the award of any degree or diploma.

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Recommendation of the Thesis Supervisors

The thesis titled "**Hybrid Grey Wolf Algorithm Development and Analysis for Effective Mult Objective Optimization**" submitted by Md Fahim Tanvir Azad, Student No: 190012128 & Rifa Bin Kawsar, Student No: 190012131 has been accepted as satisfactory in partial fulfillment of the requirements for the degree of B Sc. in Mechanical Engineering on 20th September, 2024.

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   Professor (Supervisor)
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CO-PO Mapping of ME 4800 -Theis and Project

COs	Course Outcomes (CO) Statement	(PO)	Addressed by	
CO1	Discover and Locate research problems and illustrate them via figures/tables or projections/ideas through field visit and literature review and determine/Setting aim and objectives of the project/work/research in specific, measurable, achievable, realistic and timeframe manner.	**PO2**	Thesis Book	
			Performance by research	
			Presentation and soft skill	
CO2	Design research solutions of the problems towards achieving the objectives and its application. Design systems, components or processes that meets related needs in the field of mechanical engineering	**PO3**	Thesis Book	
			Performance by research	
			Presentation and soft skill	
CO3	Review, debate, compare and contrast the relevant literature contents. Relevance of this research/study. Methods, tools, and techniques used by past researchers and justification of use of them in this work.	**PO4**	Thesis Book	
			Performance by research	
			Presentation and soft skill	
CO4	Analyse data and exhibit results using tables, diagrams, graphs with their interpretation. Investigate the designed solutions to solve the problems through case study/survey study/experimentation/simulation using modern tools and techniques.	**PO5**	Thesis Book	
			Performance by research	
			Presentation and soft skill	
CO5	Apply outcome of the study to assess societal, health, safety, legal and cultural issue and consequent possibilities relevant to mechanical engineering practice.	**PO6**	Thesis Book	
			Performance by research	
			Presentation and soft skill	
CO6	Relate the solution/s to objectives of the research/work for improving desired performances including economic, social and environmental benefits.	**PO7**	Thesis Book	
			Performance by research	
			Presentation and soft skill	
CO7	Apply moral values and research/professional ethics throughout the work, and justify to genuine referencing on sources, and demonstration of own contribution.	**PO8**	Thesis Book	
			Performance by research	
			Presentation and soft skill	
CO8	Perform own self and manage group activities from the beginning to the end of the research/work as a quality work.	**PO9**	Thesis Book	
			Performance by research	
			Presentation and soft skill	
CO9	Compile and arrange the work outputs, write the report/thesis, a sample journal paper, and present the work to wider audience using modern communication tools and techniques.	**PO10**	Thesis Book	
			Performance by research	
			Presentation and soft skill	
CO10	Organize and control cost and time of the work/project/research and coordinate them until the end of it.	**PO11**	Thesis Book	
			Performance by research	
			Presentation and soft skill	
CO11	Recognize the necessity of life-long learning in career development in dynamic real-world situations from the experience of completing this project.	**PO12**	Thesis Book	
			Performance by research	
			Presentation and soft skill	

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Name of the Supervisor:

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K-P-A Mapping of ME 4800 -Theis and Project

COs	POs	Related Ks								Related Ps							Related As				
		K1	K2	K3	K4	K5	K6	K7	K8	P1	P2	P3	P4	P5	P6	P7	A1	A2	A3	A4	A5
CO1	PO2		✓	✓	✓					✓											
CO2	PO3					✓				✓							✓				
CO3	PO4								✓	✓							✓	✓			
CO4	PO5						✓			✓		✓			✓				✓		
CO5	PO6							✓													
CO6	PO7							✓												✓	✓
CO7	PO8							✓													
CO8	PO9																				
CO9	PO10																				
CO10	PO11																				
CO11	PO12																				

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In the next step, I would like express my deep acknowledgment to my father and mother for their continued support and dedication towards my higher study.

ABSTRACT

This thesis presents an innovative approach to multi-objective optimization in industrial processes by hybridizing the Grey Wolf Optimizer (GWO) algorithm with Grey Relational Analysis (GRA). The study aims to enhance GWO's capability in handling complex, multi-faceted optimization problems. Using MATLAB software, the hybrid algorithm's effectiveness is evaluated against the standard GWO. The findings demonstrate improved efficiency and accuracy in optimization tasks, highlighting the hybrid algorithm's potential in reducing error margins and increasing convergence rates. This work's novelty lies in the unique integration of GWO with GRA, contributing significantly to optimization algorithms' theoretical understanding and practical applications. While promising, the study recognizes limitations, including its focus on specific scenarios, suggesting further scalability research and empirical validation. The enhanced GWO algorithm offers substantial practical implications for industries reliant on optimization, promising improved decision-making and process efficiency, thereby fostering continuous improvement and quality enhancement in various industrial applications.

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Nomenclature

GWO	Grey Wolf Optimizer
GRA	Grey Relational Analysis
α	Best Solution
β	Mean Solution
δ	Worst Solution
ω	Remaining Solution
PSO	Particle Swarm
GSA	Gravitational Search Algorithm
DE	Differential Evolution
FEP	Fast Evolutionary Programing
EGWO	Enchanced Grey Wolf Algorithm
D_α	Distance between the alpha wolf and a particular search agent
$X_i (0)$	Initial position of the i-th search agent
X_{max}	Upper bound of search space
X_{min}	Lower bound of search space
rand(0,1)	Random number between 0 and 1
A_1, C_1	Coefficient vectors
$X_{i'}$	New position of the i-th search agent
ABC	Artificial Bee Colony
µm	Micrometre
Ra	Surface roughness
MRR	Material Removal Rate

CHAPTER-ONE
INTRODUCTION

1.1 Introduction

The Grey Wolf Optimizer (GWO) has established itself as a significant swarm intelligence method, inspired by the social dynamics and hunting strategies of grey wolves. Renowned for its simplicity, adaptability, and effectiveness, GWO is successfully applied in diverse fields such as engineering, bioinformatics, and business. However, its standard form is primarily geared towards single-objective optimization problems, and it encounters limitations when dealing with multi-objective scenarios that are prevalent in modern complex systems.

In multi-objective optimization, several conflicting objectives must be optimized simultaneously, presenting unique challenges. The traditional GWO, while effective in single-objective scenarios, struggles with the complexity of balancing multiple conflicting objectives. This limitation has prompted the need for enhancing GWO to effectively handle multi-objective optimization, retaining its inherent strengths while enabling it to navigate the intricacies of these more complex problem spaces.

This thesis proposes an enhancement of GWO to address multi-objective optimization scenarios. This advancement aims to maintain GWO's core advantages - simplicity and efficiency - while reconfiguring its algorithmic structure to accommodate multiple, often conflicting, objectives. The enhanced GWO is expected to offer a more versatile tool for optimizing a wider array of complex, real-world problems, marking a significant advancement in the field of optimization algorithms. The successful development of this enhanced algorithm will not only contribute to the field of optimization but also open new avenues for research and application across various disciplines.

1.1.1 Optimization Algorithm:

Optimization algorithms are critical in solving complex problems across scientific and engineering disciplines. They are broadly classified into single and multi-objective optimization algorithms, each tailored to specific types of optimization challenges.

1.1.2 Single Objective Optimization Algorithm:

Single objective optimization algorithms are designed to optimize a singular metric or goal. The GWO, in its standard form, is an exemplary model in this category, offering efficient solutions for single objective optimization problems.

1.1.3 Multi Objective Optimization Algorithm:

Multi-objective optimization algorithms address problems involving several conflicting objectives. These algorithms are essential in scenarios where a balance between multiple goals is required, an area where the original GWO algorithm has limitations. Thus, enhancing GWO for multi-objective optimization represents a significant advancement in the field.

1.1.4 GWO Optimization Algorithm

The Grey Wolf Optimizer algorithm is inspired by the social hierarchy and hunting behavior of grey wolves in nature. It is structured around four primary groups - Alpha, Beta, Delta, and Omega, mirroring the leadership and decision-making hierarchy within a wolf pack. Each group within this hierarchy has a specific role during the hunting process. Fig 01 properly depicts and gives clear idea the hierarchy system in the wolves pack.

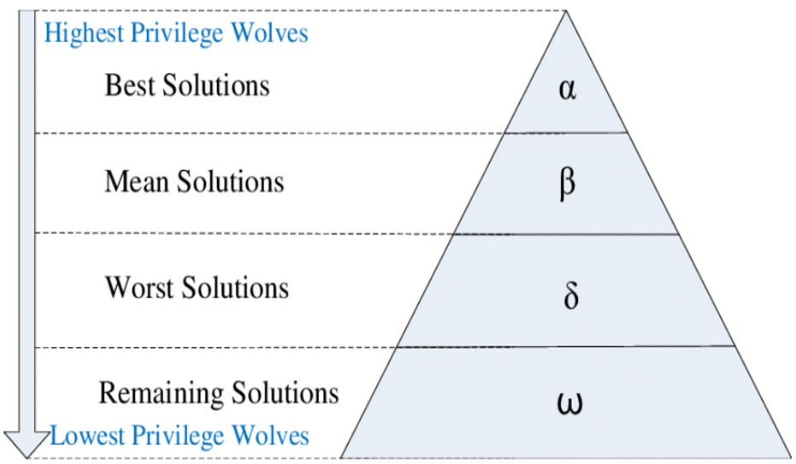


***Fig 01**: Hierarchy of grey wolf*

Alpha (α): The leader of the pack, making critical decisions and guiding the rest of the pack.

Beta (β): Subordinates to the Alpha, acting as advisors and helpers in decision-making processes.

Delta (δ): These wolves have diverse responsibilities, including scouting, sentinel, and caretaker roles.

Omega (ω): The lowest ranking in the hierarchy, usually following the lead of higher-ranked wolves.

In GWO, this social structure is translated into an algorithmic process where each wolf's position represents a potential solution, and their movements through the search space represent the exploration and exploitation of the solution space. The Alpha, Beta, and Delta wolves guide the rest of the pack (the Omegas) towards promising areas in the search space. The algorithm emulates behaviors such as chasing, encircling, and attacking prey, which metaphorically correspond to exploring, encircling, and honing in on optimal solutions.

This hierarchical approach facilitates a balance between exploration (searching for new solutions) and exploitation (refining existing solutions), which is crucial in optimization tasks. The exploration phase is akin to the wolves searching for prey, covering a wide area to locate potential targets. Once a target is identified, the exploitation phase begins, where the wolves encircle and gradually close in on the prey, akin to converging towards an optimal solution.

The dynamics of hunting and the social hierarchy of the wolves are mathematically modeled to optimize the given objective function. In the context of single-objective optimization, this approach is highly effective. However, when applied to multi-objective problems, the standard GWO algorithm requires modifications to adequately balance and optimize multiple conflicting objectives. This necessity for enhancement forms the basis for developing a more advanced version of the GWO algorithm, capable of efficiently navigating and solving complex multi-objective optimization problems..

1.1.5 Grey Relational Analysis (GRA)

GRA is an advanced analytical technique used for discerning and understanding complex interrelationships among multiple factors or variables in a given dataset. Originating from grey system theory, GRA is particularly effective in situations where information is incomplete or uncertain. In the context of optimization, GRA excels at providing a framework for considering multiple factors, which is especially valuable in multi-objective optimization scenarios. It helps in quantifying and analyzing the degree of influence that different factors have on each other, thus enabling a more informed and holistic approach to decision-making. This capability makes GRA an invaluable tool for enhancing the multi-objective optimization processes, where balancing and prioritizing competing objectives is often challenging

1.1.6 Integrating GRA with GWO

The integration of GRA with GWO algorithm presents a novel and potent approach to multi-objective optimization. This hybridization aims to synergize the strengths of both GRA and GWO, thereby effectively addressing the complexities inherent in multi-dimensional optimization tasks. By incorporating GRA into GWO, the enhanced algorithm is expected to achieve a more nuanced understanding and handling of the relationships between multiple objectives. This integration enables the algorithm to effectively navigate through a multi-objective landscape, identifying solutions that best satisfy a set of conflicting objectives.

In practice, integrating GRA with GWO could involve using GRA to analyze and rank the relative importance or influence of different objectives in a multi-objective problem. This ranking can then guide the GWO algorithm in its search process, ensuring that the wolves (solutions) not only converge towards optimal solutions but also consider the trade-offs and interdependencies among various objectives.

Furthermore, the use of GRA within the GWO framework could enhance the algorithm's ability to handle complex, real-world problems where objectives are not only multiple but also dynamic and evolving. By continuously analyzing the changing interrelationships among objectives, the hybrid algorithm can dynamically adjust its optimization strategies, maintaining relevance and effectiveness in varying conditions.

The hybrid GWO algorithm thus promises improved performance in multi-objective optimization, particularly in scenarios where the objectives are numerous, conflicting, and subject to change. It opens up new possibilities for more sophisticated and effective optimization solutions, applicable across a wide range of industries and disciplines facing complex multi-objective decision-making challenges. This advancement represents a significant stride in the field of optimization, leveraging the application of grey systems theory to enhance the capabilities of a well-established optimization method.

1.2 Research Problem Statement

1.2.1 Problem Identification

GWO faces inherent limitations when applied to multi-objective optimization problems due to its original design for single-objective tasks. Recognizing these constraints, there is potential in enhancing GWO's capabilities through hybridization with GRA. This approach aims to address the challenge of effectively managing and optimizing multiple, often conflicting, objectives in complex optimization scenarios

1.2.2 Problem Statement

The central focus of this research is the development of a hybrid GWO algorithm specifically tailored for efficient multi-objective optimization. This involves reconfiguring the standard GWO framework to integrate the multi-dimensional analysis capabilities of GRA, aiming to significantly improve its performance in multi-objective settings.

1.3 Goals and Objectives of the Study

1.3.1 Aim of the Study

The primary aim is to develop and rigorously validate a hybrid GWO algorithm. This development seeks to advance the field of multi-objective optimization by introducing a more robust and versatile optimization tool.

1.3.2 Objectives

- To integrate GRA with GWO, creating a hybrid algorithm that effectively handles multi-objective optimization tasks.
- To evaluate the performance of the hybrid algorithm in diverse multi-objective scenarios, ensuring its adaptability and efficiency.
- To benchmark the hybrid GWO algorithm against other nature-based optimization techniques, focusing on efficiency and solution quality.

1.4 Scope and Limitation of the Study

The study primarily revolves around the theoretical development and simulation-based testing of the hybrid GWO algorithm. While it aims to establish a strong theoretical foundation and demonstrate effectiveness through simulations, the primary limitation lies in its theoretical nature. Real-world applications and empirical validation of the algorithm are identified as crucial areas for future research. This limitation underscores the need for subsequent studies to translate theoretical advancements into practical applications, ensuring the algorithm's effectiveness in real-world scenarios.

1.5 Contribution of the Study

This research contributes significantly to the field of optimization by introducing a novel hybrid algorithm that combines the robustness of GWO with the analytical strength of GRA. It offers new perspectives and methodologies for tackling multi-objective optimization problems, which are increasingly prevalent in various industries. The study's contributions extend beyond theoretical advancement; it proposes practical benefits in diverse fields, from engineering and bioinformatics to business and environmental management. By addressing the challenges of multi-objective optimization with a novel approach, this research paves the way for more sophisticated, efficient, and effective solutions in complex decision-making scenarios across various sectors.

1.6 Arrangement/Organization of the Thesis:

Chapter 1: Introduction.
Chapter 2: Literature review of GWO and GRA.
Chapter 3: Methodology for developing and evaluating the hybrid algorithm.
Chapter 4: Results and analysis of the code validation.
Chapter 5: Conclusion summarizing findings, contributions, and directions for future research.

CHAPTER TWO
LITERATURE REVIEW

2.1 Introduction

This literature review provides an in-depth investigation of the recent advances in GWO and Grey Relational Analysis, which are essential tools in the optimization field by examining a range of studies emphasizing on notable advancements and breakthroughs in these fields. Extensive research has been carried out in the field of GWO to improve its effectiveness in solving both single and multi-objective optimization problems. As a result, several enhanced versions of the algorithm have been developed. These improvements tackle substantial challenges such as maximizing efficiency, finding the right balance between exploring new possibilities and exploiting known solutions, and adapting to intricate problem scenarios. However, GRA has been successfully utilized in decision-making and optimization when faced with ambiguity, demonstrating its adaptability in different multi-objective situations. The review highlights the significance of these methodologies in resolving intricate optimization problems and provides the way for exploring the possibilities of a novel hybrid algorithm.

2.2 Recent works on GWO:

For many optimization problems in various fields, GWO algorithm is considered by significant number if researchers and a upward trend can be seen in the field of studies regarding this topic.

(Mirjalili, Mirjalili & Lewis, 2014) developed the original GWO algorithm, inspired by the leadership hierarchy and hunting mechanism of grey wolves, was proposed as a new meta-heuristic technique which outperforms PSO (Particle Swarm Optimization) and GSA (Gravitational Search Algorithm) in most of the single objective optimization cases and gives competitive result while occasionally outperforming the DE (Differential Evolution) and FEP (Fast Evolutionary Programing) algorithms.

(Zhang et al., 2019) proposes the use of Grey Wolf Optimization (GWO) for optimizing the power allocation scheme in the distributed hierarchical control structure of a microgrid (MG) while considering economic factors.

(Long et al., 2019) proposed ERGWO which is a more robust version of GWO for large-scale numerical optimization problems. A parameter adjustment strategy inspired by particle swarm optimization is used by ERGWO to balance exploration and exploitation. It uses a modified position-updating equation to speed up convergence.

(Long & Xu, 2016) introduced a time-varying parameter in GWO, decreasing linearly to balance exploitation and exploration. Enhanced global convergence was achieved using the good-point-set method, showing superior performance on standard unconstrained functions.

(Kumar & Chhabra, 2016) proposed an improved version (IGWA) with a novel position updating concept, resulting in better convergence power compared to the existing GWA.

(Heidari et al.,2019) outlined several ways to take advantage of the Grey Wolf Optimizer (GWO), including using opposition-based learning, levy flight patterns, random spiral-form motions, and greedy selection. These enhancements prevent GWO from reaching local optima and make it easier to converge on solutions to difficult optimization problems.

(Seema & Kumar, 2016) offered modifications in GWA to provide a better balance between exploration and exploitation, significantly improving its performance on benchmark test functions.

(Sharma, Salgotra & Singh, 2017) developed an enhanced GWO (EGWO) based on the hunting pattern and leadership quality of grey wolves. EGWO showed better convergence compared to various algorithms like bat algorithm (BA) and firefly algorithm (FA).

(Pradhan, Roy & Pal, 2016) applied GWO to economic load dispatch problems, considering nonlinear generator characteristics. GWO proved to be an effective optimization technique for various economic load dispatch problems.

(Chen & Zhang, 2021) introduced a global search strategy in GWO to strengthen its global search ability. Adaptive weight and random search strategies were added, showing the feasibility and effectiveness of the approach.

(Martin, Marot & Bourennane, 2018) proposed a novel discrete GWO with random leader selection, increasing the probability for the main leader to be selected across iterations. This version was compared to another discrete GWO using standard test functions.

(Helmi et al., 2021) introduced a new lightweight feature selection approach. The GWO was hybridized with the gradient-based optimizer to address such an optimization problem.

2.3 Recent works of multi-objective optimization with GRA:

(Gui-wu, 2008) developed a single objective programming model for hybrid multiple attribute decision-making. This method unified different forms of numbers and determined the relation degree between alternatives and ideal points.

(Pervez et al., 2016) used grey relational analysis within the Taguchi method to optimize injection molding parameters for HDPE/TiO2 nanocomposites.

(Singh, I. Singh & Dvivedi, 2013) utilized GRA for optimizing input process parameters in drilling of metal matrix composites. The method integrated Taguchi's design of experiments with GRA to enhance output quality characteristics like thrust force, torque, and surface roughness.

(Chen, Ke & Liu, 2009) combined objective and subjective weights models in multi-attribute group decision-making using GRA. The method calculated grey relational grades to determine the weights of each expert, proving its rationality and feasibility through simulation.

(Omoniwa, 2014) applied GRA for solving Multi Criteria Robot Selection Problems (MCRSPs). The approach was validated using practical cases, demonstrating the minimal impact of the distinguishing coefficient on the GRA solution.

(Han et al., 2019) proposed an improved GRA method based on vector projections for multivariate chaotic time series prediction. The method effectively measured correlations between input and output variables, improving prediction accuracy.

(Sallehuddin, Shamsuddin & Hashim, 2008) applied GRA to multivariate time series data, focusing on identifying significant factors affecting grain crop yield in China from 1990 to

2003. The study found that the main influencing factor was the consumption of pesticide and chemical fertilizer, and GRA's performance in an ANN model was compared with multiple linear regression, showing superior forecasting accuracy.

(Xiu-Hong, 2007) developed a multi-objective decision-making model using the grey relation grade theory. This model utilized the target's grey relation grade to assign weights to indicators and employed the weighted grey relation degree of alternatives for evaluation.

(Han, Zhang, Qiu, Xu, & Ren, 2019) presented an improved Grey relational analysis (GRA) method for multivariate chaotic time series prediction. This method, based on vector projections, effectively analyzed data correlations, leading to enhanced prediction performance, especially when dealing with inexact or incomplete data.

2.4 Selection of Methods/Tools/Techniques Based on Literature Review

The literature review reveals that while there have been significant developments in both GWO and GRA independently, the potential for their hybridization, particularly in multi-objective optimization, remains largely unexplored. This presents a novel area of research with room for significant contributions

Existing studies on GWO focus on enhancing its performance for optimization problems, addressing its exploration and exploitation balance, and adapting it for discrete and global optimization. GRA studies highlight its effectiveness in decision-making, particularly in contexts with incomplete or uncertain information and multi-objective scenarios.

The hybrid algorithm aims to harness GWO's optimization capabilities with GRA's multi-dimensional analysis strength. This hybrid approach is not extensively explored, indicating a substantial room for research. The proposed hybrid GWO algorithm is expected to address the limitations of GWO in multi-objective optimization by incorporating GRA's ability to analyze and prioritize multiple conflicting objectives. This novel approach promises to enhance the performance of GWO in complex, multi-objective optimization tasks, contributing significantly to the fields of optimization and decision-making.

CHAPTER THREE
METHODOLOGY

3.1 Introduction

The methodology encompasses a thorough workflow plan that provides a detailed outline of the work process and offers a clear understanding of the tasks to be completed. The text provides an explanation of the working process and the development of MATLAB code for the GWO algorithm. It then presents an overview of how the GRA is integrated into the GWO algorithm to enhance its capabilities for multi-objective optimization.

3.2 Workflow Plan

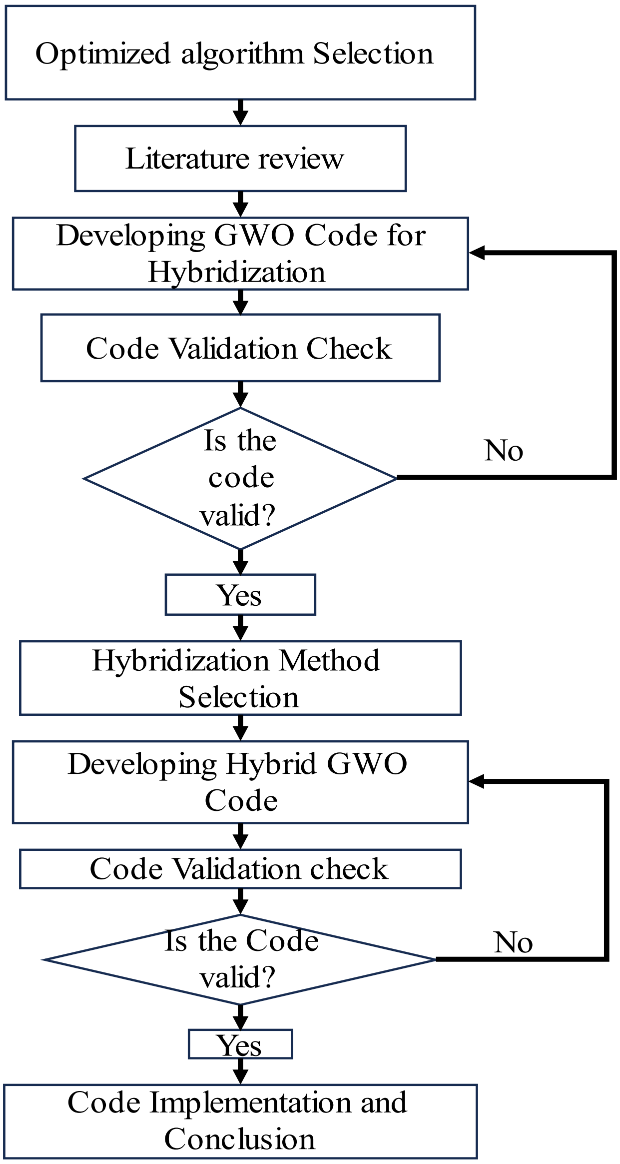


***Fig 02**: Workflow diagram*

The initial step is to choose an optimization algorithm, specifically the GWO algorithm in this case. A literature review is conducted to gain knowledge about the latest advancements in the GWO algorithm and identify potential research opportunities in the field of hybridization. The literature review revealed that the integration of GRA for multi-objective optimization scenarios has received little attention. Therefore, this integration was chosen as the focus of the research. Subsequently, a GWO code was formulated based on the initial algorithm for hybridization utilizing MATLAB software. The code was then assessed against a benchmark problem to evaluate its performance and ascertain its validity. After the validity test of single objective optimization codes, work on developing the hybrid GWO code started. Firstly, GRA calculation code was developed because it can handle multiple performance and the Grey Relational Grade gives us the ranking of best experiment where optimum results may lay. But GRA can work with only the given dataset and can't do iterations so there comes GWO and give the Optimum input parameter values. Finally, the newly developed Hybrid GWO-GRA code was validated using a benchmark multi objective optimization process of turning operation of stainless steel and was found to be satisfactory. This concludes the workflow diagram shown in Fig:02.

3.3 GWO algorithm working process

The algorithm can be visualize more clearly using this flowchart:

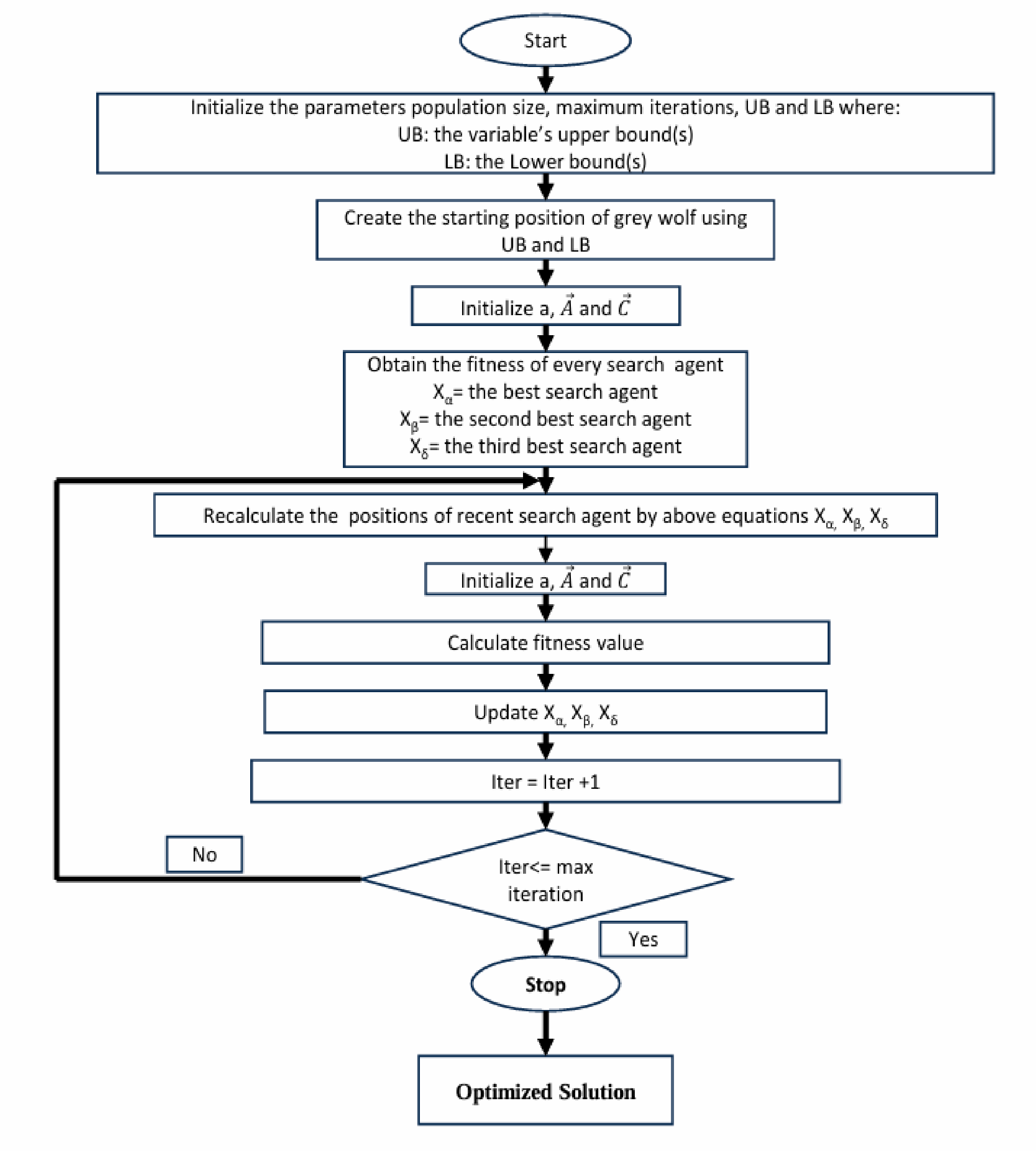


Fig 03: *GWO Algorithm Flowchart*

Detailed descriptions of the steps shown in Fig:03 is given below:

1. Initialization

- In this phase, the GWO algorithm randomly assigns initial positions to the search agents (wolves) within the specified bounds of the search space. This initialization is mathematically expressed as:

$$Xi\ (0) = Xmin + rand(0,1) \times (Xmax - Xmin)$$

Where, $X_i\ (0)$ is the initial position of the i-th search agent, X_{max} and X_{min} are the upper and lower bounds of search space, and *rand(0,1)* is a random number between 0 and 1.

2. Fitness Evaluation

- The fitness of each agent is computed using the given objective function. The fitness function depends on the problem being solved and is used to evaluate how close a given solution is to the optimum.

3. Alpha, Beta, Delta Determination

- The agents are ranked based on their fitness scores, and the top three wolves are designated as Alpha (α), Beta (β), and Delta (δ), respectively. These wolves represent the best current solutions.

4. Convergence Tracking

- The algorithm keeps track of the convergence process by storing the positions and fitness values of the Alpha, Beta, and Delta wolves at each iteration. This data is used to guide the search process and to analyze the algorithm's performance over time.

5. Position Update for Search Agents

- The positions of the search agents are updated by simulating the hunting behavior of grey wolves. The mathematical model for updating the position of the i-th search agent is given by:

$$D\alpha = |\ C1 \times X\alpha - Xi\ |\ ,\ \ Xi' = X\alpha - A1 \times D\alpha$$

And similarly, for β and δ, where A_1 and C_1 are coefficient vectors, $X_{i'}$ is the new position of the *i*-th search agent and Dα represents the distance between the alpha wolf ($X\alpha$) and a particular search agent (Xi)

- In its iterative process, the algorithm enhances the search agent positions in a continuous manner. The fitness of each agent is evaluated and the positions of the Alpha, Beta, and Delta wolves are switched every iteration. Iterations keep going until something like a certain level of fitness or the maximum number of iterations is reached.

6. Result

- GWO algorithm yields the optimized parameters and the predicted optimal solution after executing the code. The solution represents the final location of the Alpha wolf, which contains the most optimal solution discovered by the algorithm.

3.4 GWO Code Development

For the development of the GWO Code 6 part files were made using the MATLAB software and each of the files have its own working function:

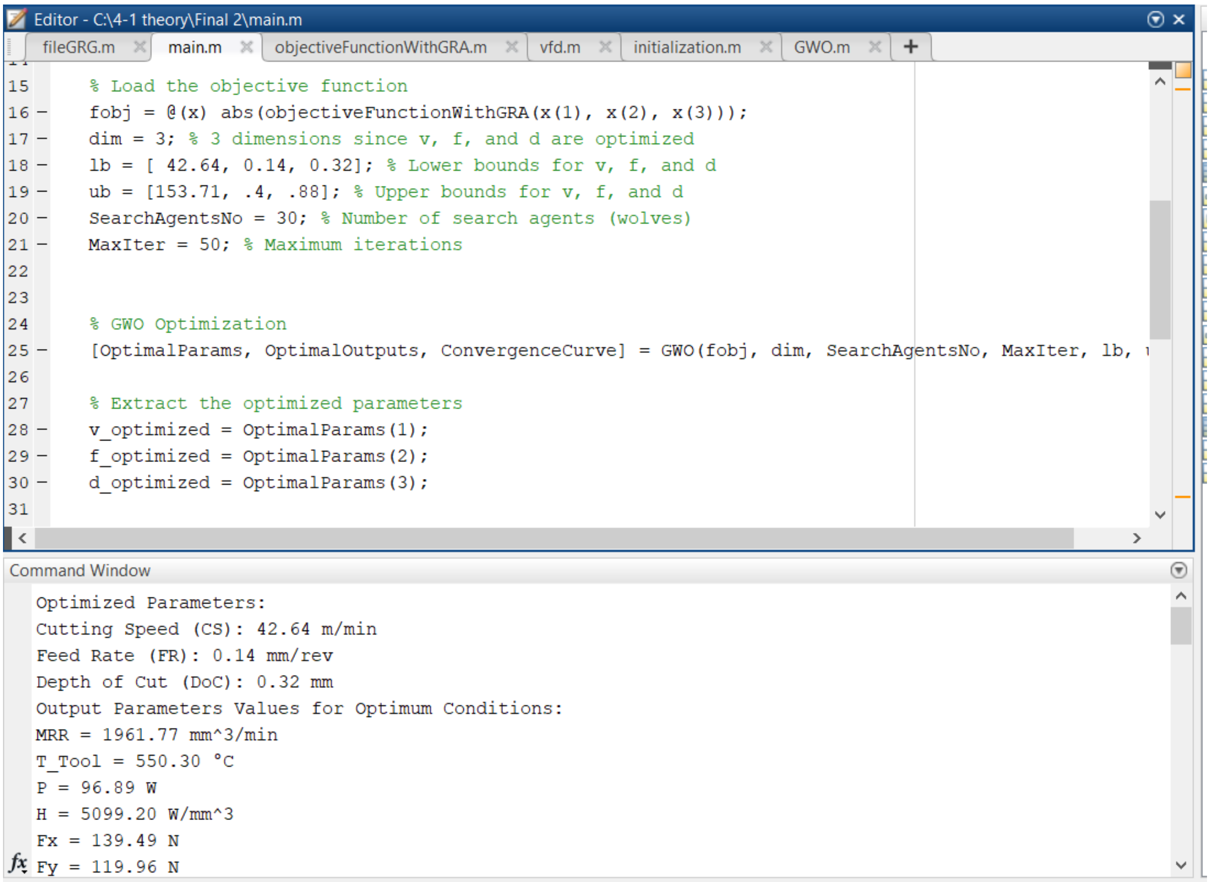


***Fig 04**: GWO MATLAB code*

fileGRG.m:

Operations:

- Loads data from 'GRA.xlsx'.
- Normalizes data to scale different parameters.
- Calculates deviations from ideal values.
- Computes Grey Relational Coefficients (GRC) and Grey Relational Grade (GRG) using weights for each parameter.
- Updates the original data table with GRC and GRG values.

GWO.m:

Operations:

- Initializes search agent positions within given bounds.
- Iterates to optimize parameters.
- Evaluates fitness for each search agent.
- Updates positions and scores for alpha, beta, and delta wolves.
- Updates search agent positions.
- Returns optimal parameters and predicted solution.

initialization.m:

Operations:

- Generates initial positions of search agents randomly within specified bounds.

main.m:(shown in Fig:04)

Operations:

- Loads the objective function from a file.
- Defines the bounds for the process parameters (lower and upper bounds for cutting speed, feed rate, depth of cut).
- Sets the number of search agents (wolves) and maximum iterations for the GWO.
- Executes the GWO optimization algorithm.
- Displays the optimized parameters and predicted roughness

objectiveFunction.m:

Operations:

- Defines the objective function that computes the optimum solution based on given parameters.

vfd.m:

Operations:

- Creates a excel file and stored the v,f,d values.
- Calculates the output parameters from the v,f,d value table .
- Creates another excel file to store the calculated output parameter values.

3.5 GRA working process

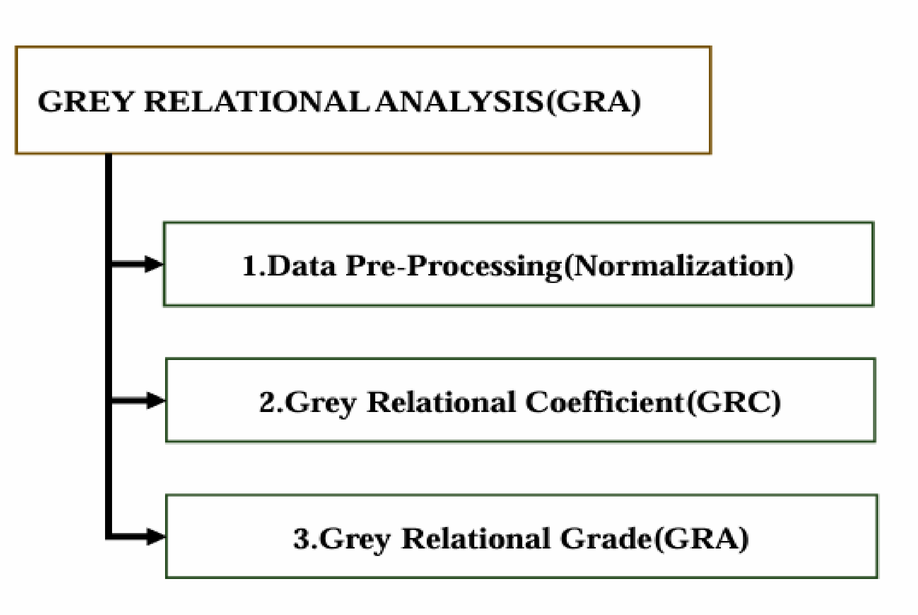


***Fig 05**: Steps in GRA*

Steps of Grey Relational Analysis (from Fig:05) discussed below in details:

1.Data pre-processing:

Data pre-processing is normally required since the range and unit in one data sequence may differ from the others. Data preprocessing is also necessary when the sequence scatter range is too large, or when the directions of the target in the sequences are different. Data pre-processing is a means of transferring the original sequence to a comparable sequence. Depending on the characteristics of a data sequence, there are various methodologies of data pre-processing available for the grey relational analysis. If the target value of original sequence is infinite, then it has a characteristic of the "higher is better". The original sequence can be normalized as follows:

$$x_i^*(k) = \frac{x_i^0(k) - min\, x_i^0(k)}{max\, x_i^0(k) - min\, x_i^0(k)} \qquad (3.5.1)$$

When the "lower is better" is a characteristic of the original sequence, then the original sequence should be normalized as follows:

$$x_i^*(k) = \frac{max\, x_i^0(k) - x_i^0(k)}{max\, x_i^0(k) - min\, x_i^0(k)} \qquad (3.5.2)$$

Where i =1 … … … m; k = 1 … … … n; m is the number of experimental data items and n is the number of parameters $x_i^0\ (k)$ denotes the original sequence $x_i * (k)$ the sequence after the data pre-processing, max $x_i^0(k)$ the largest value of $x_i^0(k)$, min $x_i^0(k)$ the smallest value of xi $0(k)$ and $x_i*(k)$ is the desired value, which is assumed 1.

2. Grey relational coefficient:

In grey relational analysis, the measure of the relevancy between two systems or two sequences is defined as the grey relational grade. When only one sequence, $x_i*(k)$, is available as the reference sequence, and all other sequences serve as comparison sequences, it is called a local grey relation measurement. After data pre-processing is carried out, the grey relation coefficient $\xi o(k)$ for the $k\ th$ performance characteristics in the $i\ th$ experiment can be expressed as:

$$\xi_i(k) = \frac{\Delta_{min}+\zeta\Delta_{max}}{\Delta_{0i}(k)+\zeta\Delta_{max}} \tag{3.5.3}$$

where, $\Delta_u(k)$ is the deviation sequence of the reference sequence and the comparability sequence.

$$\Delta_{0i}(k) = \parallel x_0^*(k) - x_i^*(k) \parallel \tag{3.5.4}$$

$$\Delta_{min} = \underset{\forall j \epsilon i}{min}\ \underset{\forall k}{min} \parallel x_0^*(k) - x_j^*(k) \parallel \tag{3.5.5}$$

$$\Delta_{max} = \underset{\forall j \epsilon i}{max}\ \underset{\forall k}{max} \parallel x_0^*(k) - x_j^*(k) \parallel \tag{3.5.6}$$

$x_0*(k)$ denotes the reference sequence and $x_i*(k)$ denotes the comparability sequence. ζ is distinguishing or identification coefficient: $\zeta \in [0,1]$ (the value may be adjusted based on the actual system requirements). A value of ζ is the smaller and the distinguished ability is the larger. $\zeta = 0.5$ is generally used.

3. Grey relational grade:

After the grey relational coefficient is derived, it is usual to take the average value of the grey relational coefficients as the grey relational grade. The grey relational grade is defined as follows:

$$\gamma_i = \frac{1}{n}\sum_{k=1}^{n}\xi_i(k) \tag{3.5.7}$$

However, in a real engineering system, the importance of various factors to the system varies. In the real condition of unequal weight being carried by the various factors, the grey relational grade in equation above was extended and defined as:

$$\gamma_i = \frac{1}{n}\sum_{k=1}^{n}w_k\,\xi_i(k) \tag{3.5.8}$$

$$\sum_{k=1}^{n}w_k = 1 \tag{3.5.9}$$

3.6 Development of Hybrid GWO for Multi-objective Optimization

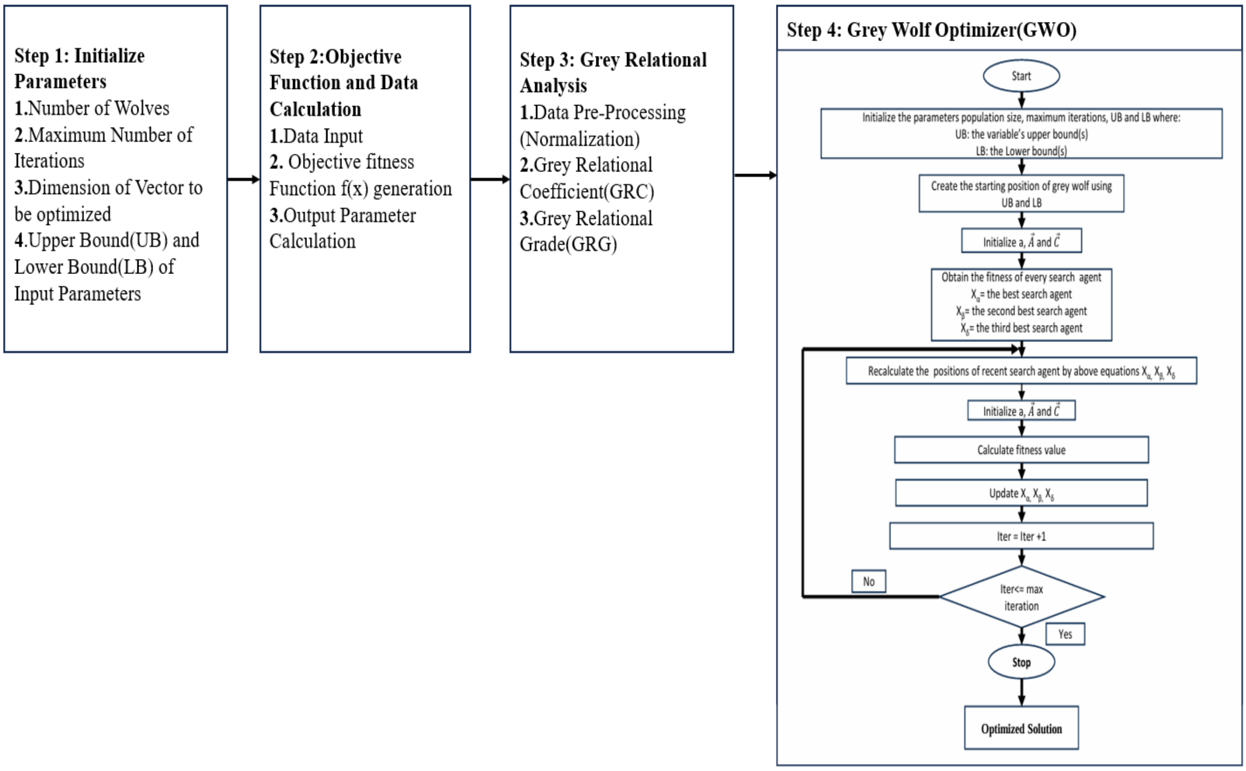


***Fig 06**: Flow Diagram of the hybridized GWO-grey relation multi-objective algorithm.*

Code Structure:

1. Defining initial Parameters of GWO (wolf no, Ib ub, Max_iter)
2. From v, f, d dataset output parameters calculated by using objective function
3. GRA (Data Pre-Processing, GRC, GRG)
4. Calculated GRG fed into GWO
5. GWO algorithm execution
6. Optimum v, f, d value

GRA can be integrated into the GWO algorithm (*Fig:06*) to enhance its multi-objective optimization capabilities and the working process of the hybrid algorithm is given below:

The Grey Wolf Optimizer Algorithm (GWO) and Grey Relational Analysis (GRA) are combined to create a hybrid approach for multi-objective optimization. GWO is a single-objective optimization algorithm that explores a defined search area to find the optimum solution, while GRA is a multi-objective analysis tool that can handle multiple output performance indexes but is limited to the provided dataset.

The hybridization process begins by defining initial parameters for GWO, such as the number of wolves and maximum iterations. Each parameter being analyzed is assigned upper and lower limits to define the search range. Data and mathematical models from previous studies are used to understand how output parameters respond to changes in the input variables.

GRA is then introduced to compute the Grey Relational Grade (GRG) for each output parameter. The empirical data is pre-processed through normalization to ensure consistent units and handle conflicting desirable levels. Grey relational coefficients are calculated, and a weighted average is taken to obtain the GRG, accounting for the different importance of each output parameter.

The calculated GRG is fed into the GWO algorithm. The GWO starts with the position of the current best search agent (alpha wolf) as an initial estimate of the optimum solution. The remaining search agents (beta, delta, and omega wolves) update their positions iteratively to move towards the alpha wolf. This process simulates the social hierarchy and hunting behavior of grey wolves, where the alpha wolf is considered the fittest solution.

The position update of the wolves is guided by the GWO equations, which determine the movement direction and step size of each wolf. The GWO algorithm explores the search space by balancing exploration (searching for new solutions) and exploitation (refining existing solutions).

As the GWO optimizes with respect to the GRG, which incorporates all the output parameters, the multi-objective optimization requirement is fulfilled. The algorithm continues until the maximum number of iterations is reached, resulting in an optimal solution that balances the multiple objectives based on their assigned weights in the GRA.

CHAPTER FOUR
EXPERIMENTAL DESIGN

4.1 Design of Experiment

Experimental design is widely used for controlling the effects of parameters in many processes. We worked with the turning operation of stainless steel for the assessment of the developed GWO algorithm's multi objective optimization capabilities. In this study, total three parameters input parameter are studied. They are cutting speed (v), feed rate (f) and depth of cut (d). In addition to that also seven output parameters: Material Removal Rate (MRR), Peak Tool Temperature (T_Tool), Heat Rate (H), Power (P), Force (X), Force (Y), Roughness (Ra) are taken into consideration. For all the input parameters, upper bounds and lower bounds of data are used.

4.2 Experimental Setup

Initial dataset for the input cutting parameters were taken from (M. H. Tanvir et al.,2020). After that the GWO algorithm prepares its own dataset according to the initially given dataset. The dataset we worked with is given below:

No	v	f	d
1	88.96	0.11	0.51
2	122.65	0.32	0.62
3	33.19	0.2	0.82
4	76.23	0.21	0.53
5	58.95	0.27	0.83
6	52.9	0.16	0.67
7	63.33	0.29	0.33
8	81.03	0.18	0.85
9	86.71	0.3	0.71
10	102.49	0.33	0.88
11	89.2	0.17	0.42
12	118.75	0.25	0.4
13	65.35	0.33	0.85
14	140.18	0.25	0.72
15	45.69	0.16	0.36
16	117.11	0.28	0.75
17	89	0.32	0.75
18	104.7	0.28	0.84
19	58.24	0.39	0.72
20	64.65	0.3	0.29
21	131.58	0.38	0.23

22	150.19	0.18	0.24
23	77.46	0.18	0.34
24	119.54	0.35	0.46
25	139.99	0.15	0.81
26	142.01	0.19	0.63
27	52.09	0.39	0.63
28	46.98	0.24	0.8
29	41.51	0.34	0.29
30	140.18	0.33	0.48
31	43.57	0.37	0.65
32	89.42	0.31	0.87
33	149.04	0.34	0.64
34	101.86	0.24	0.34
35	119.49	0.22	0.77
36	77.69	0.38	0.46
37	118.89	0.26	0.78
38	135.35	0.4	0.54
39	44.68	0.32	0.81
40	125.96	0.31	0.74
41	152.48	0.17	0.64
42	125.74	0.39	0.4
43	73.79	0.16	0.26
44	130.31	0.3	0.39
45	54.11	0.25	0.35
46	92.39	0.21	0.39
47	143.56	0.38	0.45
48	75.26	0.29	0.72
49	74.61	0.15	0.64
50	57.09	0.31	0.33

Table 1: Input cutting parameter dataset

	Parameters [Unit]	Lower Bound	Upper Bound
v:	Cutting Speed (v) [m/min]	33.19	153.71
f:	Feed rate (f) [mm/rev]	0.11	0.4
d:	Dept of Cut (d) [mm]	0.23	0.88

Table 2: Process parameters with their lower bound and upper bound values

GWO algorithm was employed to optimize cutting parameters. The initial data (Table 1) consisted of 50 entries, each containing three cutting parameters: cutting speed (v), feed rate (f), and depth of cut (d). Table 2 further clarifies the lower and upper limits for each parameter. GWO leverages this initial data to define a search space for optimization. It presumably doesn't alter the provided data directly. Instead, it treats this data as a reference and establishes a population of potential solutions within the designated boundaries (Tables 1 & 2). These potential solutions then undergo an iterative process of competition and collaboration to identify the optimal combination of cutting parameters based on a predefined objective function

4.3 Experimental results:

For Grey Wolf optimization (GWO) process, quadratic equations for fitness function is used. Empirical equations that are developed to calculate seven output parameters are given below:

$$\boldsymbol{MRR} = \exp\,(4.77 + 0.023v + 8.22f + 3.39 \times d - 0.0014vf - 0.00071vd - 0.053fd - 5.7010^{-5}v^2 - 7.58f^2 - 1.29d^2 \tag{4.3.1}$$

$$\boldsymbol{T_{tool}} = \exp\,(2.39 + 0.016v + 4.84f + 3.31d - 0.0035vf - 0.0025vd - 0.61fd - 2.110^{-5} \times v^2 - 2.47f^2 - 0.65d^2 \tag{4.3.2}$$

$$\boldsymbol{H} = \exp\,(8.72 + 0.022v - 8.53f + 0.031d - 1.3010^{-5}vf - 3.32 \times 10^{-6}vd - 2.0 \times 10^{-5}f^2 - 5.8 \times 10^{-5}v^2 + 8.56f^2 + 0.0082d^2 \tag{4.3.3}$$

$$\boldsymbol{P} = \exp\,(6.18 - 0.0044v - 0.057f + 0.85d - 0.0021vf - 0.0011vd - 1.14fd + 3.9 \times 10^{-5}v^2 + 3.35f^2 - 0.038d^2 \tag{4.3.4}$$

$$\boldsymbol{F_x} = \exp\,(3.77 - 0.0082v + 4.33f + 3.06d - 0.0035vf - 0.0011vd - 0.066fd + 4.5 \times 10^{-5}v^2 - 2.15 \times f^2 - 0.67d^2 \tag{4.3.5}$$

$$\boldsymbol{F_y} = \exp\,(3.40 - 0.00064v + 3.56f + 3.46d - 0.0017vf - 0.0023vd - 0.75fd + 1.6 \times 10^{-5}v^2 - 2.65f^2 - 0.91d^2 \tag{4.3.6}$$

$$\boldsymbol{R_a} = \exp\,(-1.34 + 0.0035v + 7.95f + 1.83d + 0.014vf + 0.0087vd + 0.48fd - 6.17 \times 10^{-5}v^2 - 12.39 \times f^2 - 1.42d^2 \tag{4.3.7}$$

Trial 01 Results:

Table 3: Weightage used in trial 01

MRR	P	H	T_Tool	F_x	F_y	R_a
0.2	0.1	0.1	0.2	0.1	0.1	0.2

Table 4: Results obtained for trial 01

Optimized Parameters	
Cutting Speed(v)	42.64 m/min
Feed rate(f)	0.14 mm/rev
Depth of Cut(d)	0.32 mm
Output Parameters Value for Optimum Condition	
MRR(material removal rate)	**1961.77 mm^3/min**
T_Tool(tool temperature)	550.30 °C
P(power)	96.89 W
H(heat generation rate)	5099.20 W/mm^3
F_x(force in X direction)	139.49 N
F_y(force in Y direction)	119.96 N
R_a(surface roughness)	**1.26 µm**

Trial 01 focused on optimizing cutting parameters using the GWO algorithm. Weightages (Table 3) were assigned to various output parameters such as Material Removal Rate (MRR), Power Consumption (P), Heat Generation Rate (H), Tool Temperature (T_Tool), Cutting Force in X direction (Fx), Cutting Force in Y direction (Fy), and Surface Roughness (Ra). These weightages reflect the relative importance of each parameter in achieving the desired machining outcome.

The GWO algorithm gives an optimal result of cutting parameters (Table 4). The cutting speed (v) was set at 42.64 m/min, feed rate (f) at 0.14 mm/rev, and depth of cut (d) at 0.32 mm. These parameters resulted in specific values for the output parameters as shown in Table 4. Notably, the achieved Material Removal Rate (MRR) was 1961.77 mm^3/min, while Tool Temperature (T_Tool) remained at a moderate level of 550.30 °C. Power Consumption (P) was measured at 96.89 W, and Heat Generation Rate (H) was 5099.20 W/mm^3. The cutting forces (Fx and Fy) were within acceptable ranges (139.49 N and 119.96 N respectively), and the achieved surface roughness (Ra) was 1.26 µm.

Further analysis was conducted on the forthcoming to explore the impact of varying the weightages (Table 3) on the optimal solution and resulting output parameters. This would provide valuable insights into achieving the desired balance between machining efficiency, tool life, surface quality, and other critical factors.

Trial 02 Results:

Table 5: Weightage used in trial 02

MRR	P	H	T_Tool	F_x	F_y	R_a
0.4	0.1	0.1	0.1	0.1	0.1	0.1

Table 6: Results obtained for trial 02

Optimized Parameters	
Cutting Speed(v)	152.90 m/min
Feed rate(f)	0.36 mm/rev
Depth of Cut(d)	0.86 mm
Output Parameters Value for Optimum Condition	
MRR(material removal rate)	49981.51
T_Tool(tool temperature)	1019.44 °C
P(power)	1689.25 W
H(heat generation rate)	6553.99 W/mm^3
F_x(force in X direction)	762.24 N
F_y(force in Y direction)	540.19 N
R_a(surface roughness)	1.92 µm

Trial 02 further demonstrates the GWO algorithm's effectiveness in optimizing cutting parameters based on user-defined priorities. In this trial, the weightage for Material Removal Rate (MRR) was significantly increased to 40% (Table 5), compared to 20% in Trial 01. This shift reflects a prioritization of maximizing material removal efficiency.

The Hybrid GWO algorithm responded to the adjusted weightages by identifying a new set of optimal cutting parameters (Table 6). The cutting speed (v) increased considerably to 152.90 m/min, feed rate (f) rose to 0.36 mm/rev, and depth of cut (d) became significantly larger at 0.86 mm. As expected with these more aggressive settings, the achieved Material Removal Rate (MRR) soared to 49981.51 mm^3/min, a substantial increase compared to Trial 01.

However, these increased cutting parameters also resulted in trade-offs. Tool Temperature (T_Tool) rose to 1019.44 °C, Power Consumption (P) jumped to 1689.25 W, and Heat Generation Rate (H) became higher at 6553.99 W/mm^3. Cutting Forces (Fx and Fy) also increased to 762.24 N and 540.19 N respectively. Surface Roughness (Ra) remained acceptable at 1.92 µm.

This comparison between Trial 01 and Trial 02 highlights the impact of weightage selection on the optimization process. By prioritizing Material Removal Rate, a significant increase in material removal efficiency can be achieved but at the expense of increased tool wear, power consumption, and cutting forces.

For practical applications, selecting weightages should consider the specific machining requirements. If rapid material removal is essential, prioritizing MRR like Trial 02 might be suitable. However, if tool life, power consumption, or surface quality are critical factors, a more balanced weightage distribution, like Trial 01, may be preferable.

Trial 03 Results:

Table 7: Weightage used in trial 03

MRR	P	H	T_Tool	F_x	F_y	R_a
0.1	0.1	0.1	0.1	0.1	0.1	0.4

Table 8: Results obtained for trial 03

Optimized Parameters	
Cutting Speed(v)	33.19 m/min
Feed rate(f)	0.11 mm/rev
Depth of Cut(d)	0.23 mm
Output Parameters Value for Optimum Condition	
MRR(material removal rate)	1077.71 mm^3/min
T_Tool(tool temperature)	523.10 °C
P(power)	59.15 W
H(heat generation rate)	5213.48 W/mm^3
F_x(force in X direction)	103.88 N
F_y(force in Y direction)	86.56 N
R_a(surface roughness)	Ra = 0.68 µm

In this trial, the weightage for Surface Roughness (Ra) was significantly increased to 40% (Table 7), compared to a weighting of only 10% in both Trial 01 and Trial 02. This adjustment reflects a clear focus on achieving the smoothest possible surface finish.

The GWO algorithm responded to the new priorities by selecting a very conservative set of cutting parameters (Table 8). The cutting speed (v) was set to the minimum allowable value of 33.19 m/min, feed rate (f) was also set to the minimum value of 0.11 mm/rev, and depth of cut (d) was kept at the lowest possible level of 0.23 mm. As expected with these cautious settings, the achieved Material Removal Rate (MRR) was the lowest across all three trials at 1077.71 mm^3/min.

However, the trade-off for this low material removal rate was a significant improvement in surface quality. Surface Roughness (Ra) achieved an excellent value of 0.68 µm, the lowest recorded across all trials. Additionally, Tool Temperature (T_Tool) remained moderate at

523.10 °C, Power Consumption (P) was minimal at 59.15 W, and Cutting Forces (Fx and Fy) were the lowest observed (103.88 N and 86.56 N respectively). Heat Generation Rate (H) was also the lowest at 5213.48 W/mm^3.

The results from Trial 03 clearly demonstrate the impact of weightage on achieving a desired surface finish. By prioritizing Surface Roughness, we achieved an exceptional level of smoothness but at the cost of significantly reduced machining efficiency.

Trial 04 Results:

Table 9: Weightage used in trial 04

MRR	P	H	T_Tool	F_x	F_y	R_a
0.2	0.1	0.1	0.1	0.1	0.1	0.3

Table 10: Results obtained for trial 04

Optimized Parameters	
Cutting Speed(v)	39.76 m/min
Feed rate(f)	0.13 mm/rev
Depth of Cut(d)	0.30 mm
Output Parameters Value for Optimum Condition	
MRR (material removal rate)	**1665.62 mm^3/min**
T_Tool(tool temperature)	544.06 °C
P(power)	85.05 W
H(heat generation rate)	5159.95 W/mm^3
F_x(force in X direction)	129.75 N
F_y(force in Y direction)	110.88 N
R_a(surface roughness)	0.86 µm

Trial 04 further emphasizes the influence of weightage selection on the GWO algorithm's optimization process. Here, the weightage for Material Removal Rate (MRR) was increased to 20% (Table 9) compared to 10% in Trial 03. This shift indicates a desire for a slight improvement in material removal efficiency while still prioritizing other output parameters.

The Hybrid GWO algorithm again responded by selecting a set of cutting parameters (Table 10) that achieved a balance between material removal and other considerations. The cutting speed (v) increased slightly to 39.76 m/min compared to Trial 03, feed rate (f) also rose modestly to 0.13 mm/rev, and depth of cut (d) became slightly larger at 0.30 mm. These parameter adjustments resulted in a noticeable increase in Material Removal Rate (MRR) to 1665.62 mm^3/min, which is significantly higher than Trial 03.

However, these changes were moderate enough to maintain good results in other aspects. Tool Temperature (T_Tool) remained moderate at 544.06 °C, Power Consumption (P) was still low at 85.05 W, and Cutting Forces (Fx and Fy) were within acceptable ranges (129.75 N and 110.88 N respectively). Interestingly, Surface Roughness (Ra) also showed a slight improvement to 0.86 µm compared to Trial 03.

The results from Trial 04 highlight the possibility of achieving a compromise between multiple conflicting objectives. By carefully selecting weightages, the hybrid algorithm achieved a modest increase in material removal rate while maintaining good tool life, low power consumption, acceptable cutting forces, and even a slight improvement in surface roughness compared to the extreme prioritization on surface roughness in Trial 03.

Trial 05 Results:

Table 11: Weightage used in trial 05

MRR	P	H	T_Tool	F_x	F_y	R_a
0.3	0.1	0.1	0.1	0.1	0.1	0.2

Table 12: Results obtained for trial 05

Optimized Parameters	
Cutting Speed(v)	133.46 m/min
Feed rate(f)	0.36 mm/rev
Depth of Cut(d)	0.81 mm
Output Parameters Value for Optimum Condition	
MRR(material removal rate)	**31843.40 mm^3/min**
T_Tool(tool temperature)	909.05 °C
P(power)	1355.53 W
H(heat generation rate)	5949.89 W/mm^3
F_x(force in X direction)	658.53 N
F_y(force in Y direction)	489.93 N
R_a(surface roughness)	**1.45 µm**

Trial 05 reinforces the concept of how weightage allocation steers the GWO algorithm towards specific cutting parameter optimizations. Here, the weightage for Material Removal Rate (MRR) was assigned the highest value of 30% (Table 11), similar to Trial 01. This prioritizes maximizing material removal efficiency.

The GWO algorithm responded accordingly, selecting a set of cutting parameters (Table 12) that significantly increase material removal rate. The cutting speed (v) reached a high value of 133.46 m/min, feed rate (f) increased to 0.36 mm/rev, and depth of cut (d) became considerably larger at 0.81 mm. As anticipated with these aggressive settings, the achieved Material Removal Rate (MRR) soared to 31843.40 mm3/min, the highest across all five trials.

Similar to Trial 01, this trial focuses on maximizing material removal comes with trade-offs. Tool Temperature (T_Tool) rose significantly to 909.05 °C, Power Consumption (P) jumped to a substantial 1355.53 W, and Heat Generation Rate (H) became the highest at 5949.89 W/mm3. Cutting Forces (Fx and Fy) also increased noticeably to 658.53 N and 489.93 N respectively. Surface Roughness (Ra) remained acceptable at 1.45 µm, but it is slightly higher compared to Trial 01.

The comparison between Trial 01 and Trial 05 showcases the impact of slightly altered weightage distribution even when Material Removal Rate remains the top priority. In Trial 05, a higher weightage for MRR resulted in a more aggressive parameter selection compared to Trial 01, leading to a further increase in material removal but also slightly higher tool wear, power consumption, and cutting forces.

Overall, the five trials effectively demonstrate then novel Hybrid GWO algorithm's versatility in optimizing cutting parameters based on different conflicting priorities. By carefully selecting weightages, industries can achieve the desired balance between various machining objectives, allowing for a tailored approach to different machining requirements.

4.3.1 Validation of the Developed Hybrid Algorithm:

Table 13: Comparison Between Hybrid GWO and Hybrid Whale Optimization Algorithm for Trial 01

Parameters	Whale	GWO	Deviation (%)
v	42.64	42.64	0
f	0.14	0.14	0
d	0.32	0.32	0
MRR	2004.85	1961.77	2.19
T_Tool	550.7	550.3	0.072
P	98.9	96.89	2.074
H	5218.5	5099.2	2.33
Fx	139.8	139.49	0.22
Fy	116.3	119.96	3.05
Ra	1.14	1.26	**9.52**

Table 14: Comparison Between Hybrid GWO and Hybrid Whale Optimization Algorithm for Trial 02

Parameters	Whale	GWO	Deviation (%)
v	153.71	152.90	0.52
f	0.40	0.36	11.11
d	0.88	0.86	2.32
MRR	52130.32	49981.51	4.29
T_Tool	1057.2	1019.44	3.7
P	1994.4	1689.25	**18.06**
H	6577.4	6553.99	0.37
Fx	844	762.24	10.72
Fy	401.9	540.19	**25.60**
Ra	1.85	1.92	3.64

In this section the validity of the developed Hybrid Grey Wolf Optimization (GWO) algorithm was assessed by comparing its results with a Hybrid Whale Optimization Algorithm (Tables 13 & 14). Both algorithms tackled the same multi-objective optimization process for cutting parameters using a similar dataset in Trials 01 and 02.

The key parameters analyzed include cutting speed (v), feed rate (f), depth of cut (d), and the resulting output parameters like Material Removal Rate (MRR), Tool Temperature (T_Tool), Power Consumption (P), Heat Generation Rate (H), Cutting Forces in X and Y directions (Fx and Fy), and Surface Roughness (Ra).

In Trial 01, both algorithms yielded very similar results for most parameters (v, f, d, MRR, T_Tool, H, Fx) with deviations below 3%. The most significant difference was observed in Surface Roughness (Ra) at 9.52%.

Trial 02 presented a similar pattern. Most parameters exhibited minimal deviations (less than 5%) between the two algorithms. However, Power Consumption (P) and Force in Y direction (Fy) showed slightly higher deviations of 18.06% and 25.60% respectively.

Considering the minimal deviations across most parameters in both trials, the Hybrid GWO algorithm appears to produce valid solutions. Even the higher deviations in Surface Roughness (Trial 01) and Power Consumption/Force Y-direction (Trial 02) remain within an acceptable range for practical applications. This suggests that the Hybrid GWO algorithm performs well when compared to the Hybrid Whale Optimization Algorithm for these specific scenarios.

4.3.2 Effect of Weightage on Surface Roughness and Material Removal Rate:

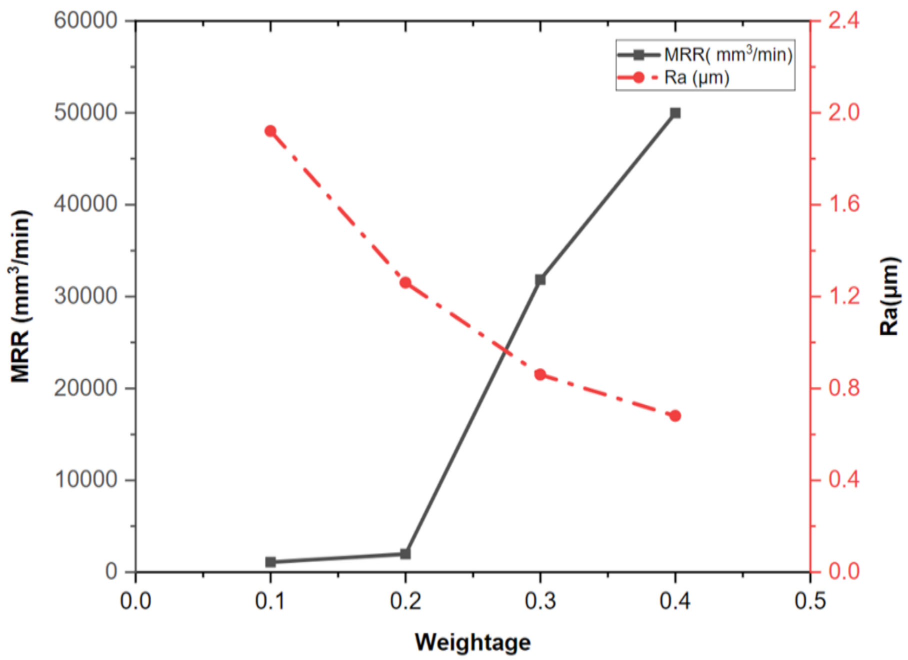


***Fig 07**: MRR and Ra vs Weightage*

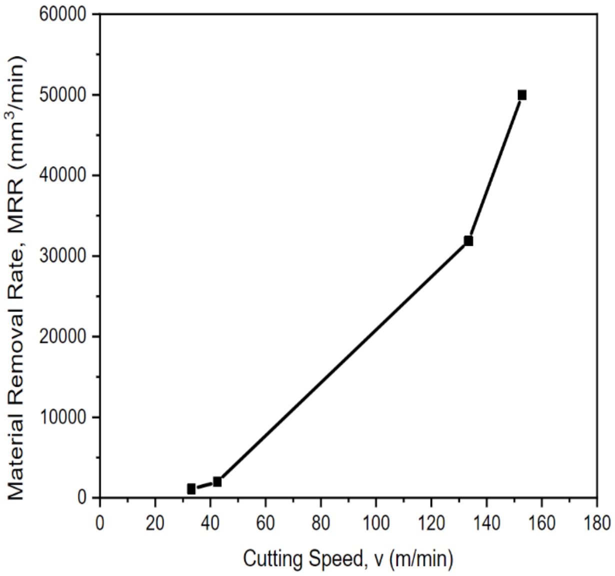
In the graph (Figure 07), it appears that increasing the weightage for Material Removal Rate (MRR) results in a proportional increase in MRR, while Surface Roughness (Ra) decreases. This relationship can be expressed as:

- **MRR is proportional to weightage**
- **Ra is inversely proportional to weightage**

This implies that prioritizing MRR through higher weightage will lead to a higher material removal rate but at the expense of a rougher surface finish. Conversely, prioritizing a smoother surface finish by assigning a higher weightage to Ra will necessitate lower material removal rates.

4.3.3 Effect of Input Cutting Parameters on Surface Roughness and Material Removal Rate:

- **Effect on Material Removal Rate:**



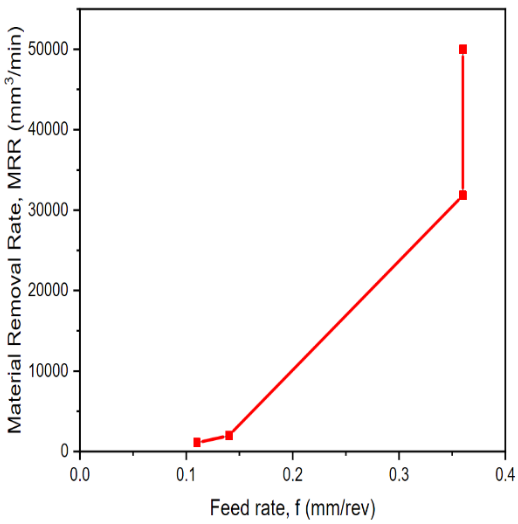


***Fig 08**: Material Removal Rate vs Cutting Speed* ***Fig 09**: Material Removal Rate vs Feed rate*

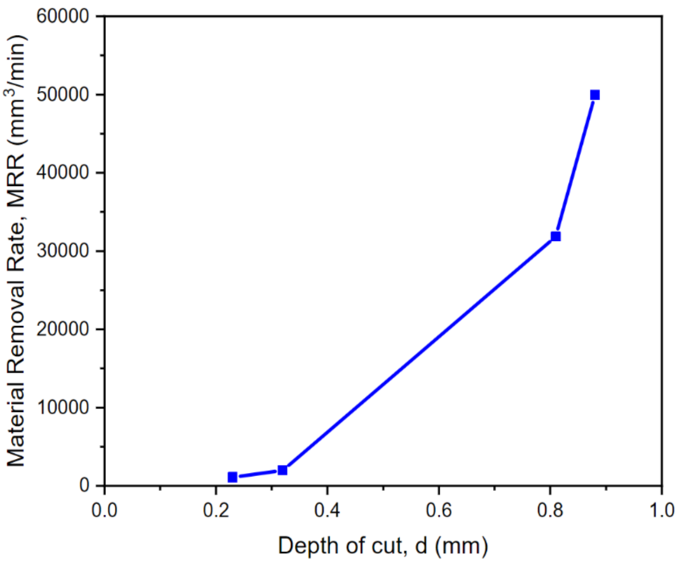


***Fig 10**: Material Removal Rate vs Depth of Cut*

Based on the graphs (Fig 08, Fig 09, Fig 10) in this section, it appears that increasing the weightage for Material Removal Rate (MRR) generally leads to an increase in MRR and a decrease in Surface Roughness (Ra). This suggests a positive correlation between MRR and weightage, and a negative correlation between Ra and weightage. In other words, prioritizing MRR with higher weightage settings tends to result in higher material removal rates but rougher surfaces, while prioritizing Ra with lower weightage settings tends to result in smoother surfaces but at the cost of lower material removal rates.

- **Effect on Surface Roughness:**

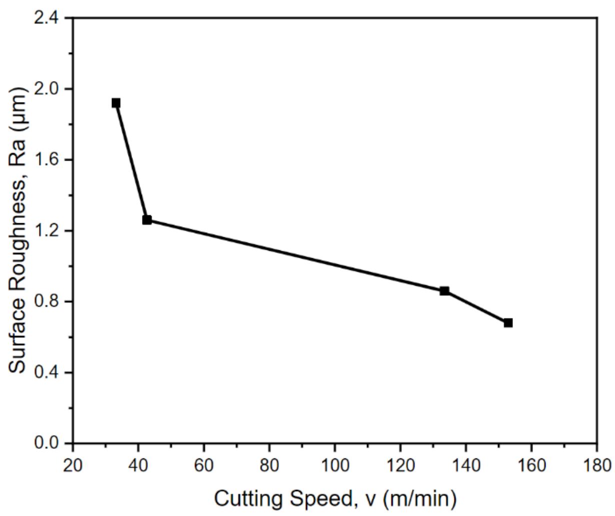


Fig 11: *Surface Roughness vs Cutting Speed*

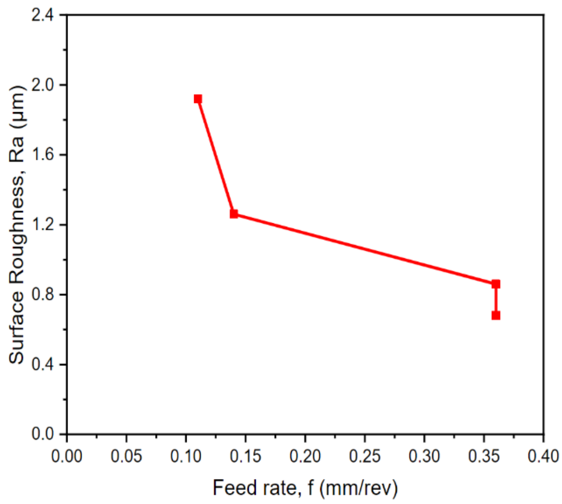


Fig 12: *Surface Roughness vs Feed Rate*

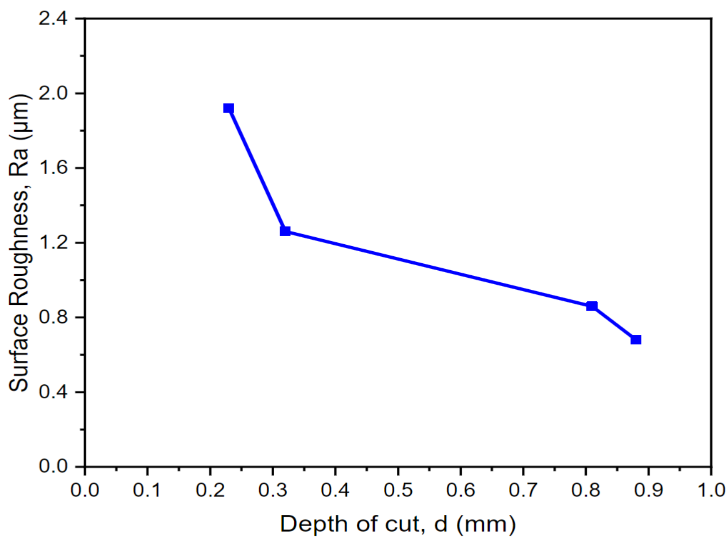


Fig 13: *Surface Roughness vs Depth of Cut*

The graphs (Fig 11, Fig 12, Fig 13) show a clear trend: as the weightage for material removal rate (MRR) is gradually increased, the MRR itself increases, while surface roughness (Ra) decreases.

This suggests a proportional relationship between MRR and weightage, and an inverse proportional relationship between Ra and weightage. In other words, prioritizing MRR with higher weightage settings leads to higher material removal rates but rougher surfaces. Conversely, prioritizing Ra with lower weightage settings results in smoother surfaces but at the cost of lower material removal rates.

4.3.4 Effect of Changing the Maximum Number of Iteration:

Table 15: Output results for Maximum Iteration No 500

Optimized Parameters	
Cutting Speed(v)	42.64 m/min
Feed rate(f)	0.14 mm/rev
Depth of Cut(d)	0.32 mm
Output Parameters Value for Optimum Condition	
MRR(material removal rate)	1961.77 mm^3/min
T_Tool(tool temperature)	550.30 °C
P(power)	96.89 W
H(heat generation rate)	5099.20 W/mm^3
F_x(force in X direction)	139.49 N
F_y(force in Y direction)	119.96 N
R_a(surface roughness)	1.26 µm

Table 16: Output results for Maximum Iteration No 5000

Optimized Parameters	
Cutting Speed(v)	42.64 m/min
Feed rate(f)	0.14 mm/rev
Depth of Cut(d)	0.32 mm
Output Parameters Value for Optimum Condition	
MRR(material removal rate)	1961.77 mm^3/min
T_Tool(tool temperature)	550.30°C
P(power)	96.89 W
H(heat generation rate)	5099.20 W/mm^3
F_x(force in X direction)	139.49 N
F_y(force in Y direction)	119.96 N
R_a(surface roughness)	1.26 µm

Increasing the maximum iteration number from 50 to 500 and then to 5,000 did not affect the final optimized cutting parameters (Table 15 & 16). This suggests that the Grey Wolf

Optimization (GWO) algorithm likely converged on the optimal solution within the initial 50 iterations for this particular scenario.

In optimization algorithms, convergence refers to the iterative process reaching a solution that meets the specified criteria. Here, the algorithm appears to have efficiently identified the optimal combination of cutting speed, feed rate, and depth of cut to achieve the desired machining outcomes within the first 50 iterations. There were no further improvements observed by increasing the maximum iterations to 500 or 5,000.

This is a positive finding, as it indicates that the Hybrid GWO algorithm can be computationally efficient for this specific optimization task. It was able to find the best solution without requiring a large number of iterations.

4.3.5 Effect of Changing the Number of Search Agents:

Table 17: Output Parameter for Search Agent No 30

Optimized Parameters	
Cutting Speed(v)	42.64 m/min
Feed rate(f)	0.14 mm/rev
Depth of Cut(d)	0.32 mm
Output Parameters Value for Optimum Condition	
MRR(material removal rate)	1961.77 mm^3/min
T_Tool(tool temperature)	550.30 °C
P(power)	96.89 W
H(heat generation rate)	5099.20W/mm^3
F_x(force in X direction)	139.49 N
F_y(force in Y direction)	119.96 N
R_a(surface roughness)	1.26 µm

Table 18: Output Parameter for Search Agent No 230

Optimized Parameters	
Cutting Speed(v)	42.64m/min
Feed rate(f)	0.18mm/rev
Depth of Cut(d)	0.32 mm
Output Parameters Value for Optimum Condition	
MRR(material removal rate)	2433.96 mm^3/min
T_Tool(tool temperature)	561.85 °C
P(power)	111.40W
H(heat generation rate)	4098.22 W/mm^3
F_x(force in X direction)	158.97 N
F_y(force in Y direction)	131.32 N
R_a(surface roughness)	1.51 µm

The tables (17 & 18) show the impact of varying search agent numbers in the Grey Wolf Optimization (GWO) algorithm.

While the cutting speed (v) remained the same (42.64 m/min) in both trials (30 and 230 search agents), other parameters differed:

- Feed rate (f) increased from 0.14 mm/rev to 0.18 mm/rev with 230 search agents.
- Material Removal Rate (MRR) rose from 1961.77 mm^3/min to 2433.96 mm^3/min with more search agents.
- Tool Temperature (T_Tool) and Power Consumption (P) also increased slightly.
- Cutting Forces (Fx and Fy) exhibited a noticeable rise.
- Surface Roughness (Ra) increased from 1.26 µm to 1.51 µm.

These changes suggest that a higher number of search agents might lead to a focus on maximizing material removal rate (MRR) at the expense of other factors like surface finish and tool wear. However, a more definitive conclusion requires further investigation

CHAPTER FIVE
CONCLUSION AND RECOMMENDATION

5.1 Summary of the Work:

This section provides a brief overview of the major activities that were carried out, as outlined in the thesis. This involves the development and evaluation of a hybrid Grey Wolf Optimizer (GWO) algorithm, which integrates Grey Relational Analysis (GRA) to improve multi-objective optimization. The study encompassed the development and evaluation of the hybrid algorithm through code validation on benchmark problem, and the comparative examination of its performance in relation to standard GWO and alternative optimization techniques.

5.2 Conclusion

- The study's main discoveries involve the effective improvement of GWO's capacity in multi-objective optimization situations.
- The hybrid GWO Algorithm exhibited enhanced efficacy and precision in optimization endeavors.
- It is found that if one output parameter is prioritized then it affects other parameters depending on the preprocessing condition of that particular parameter.
- If any changes occurs in the output parameters weightage than it significantly impacts the cutting parameters (v,f,d).
- The findings have practical implications as they provide a reliable instrument for making complex decisions and improving process efficiency across different industries.

5.3 Recommendation:

Subsequent research should prioritize the empirical verification and practical implementation of the hybrid algorithm to ascertain its usefulness in real-life scenarios. For this the developed hybrid algorithm needs to be validated against other existing nature-based algorithms of similar criteria.

Further research can investigate the scalability and impact of varying search agent (wolf) numbers on optimization outcomes.

Further investigation can be conducted to explore additional hybridization strategies that combine GWO with other optimization or analytical methods in order to enhance optimization capabilities.

APPENDIX:

Trial 01:

weights = [0.2 0.1 0.1 0.2 0.1 0.1 0.2];

No	MRR	T_Tool	H	P	F_x	F_Y	R_a	GRG
1	0.36567	0.37173	0.57654	0.37136	0.35266	0.37041	0.37692	0.38992
2	0.66487	0.60996	0.42803	0.57861	0.47955	0.51951	0.65837	0.58408
3	0.37212	0.40552	0.36552	0.50865	0.59207	0.63503	0.42677	0.46132
4	0.38033	0.38161	0.41779	0.38043	0.38117	0.39768	0.44083	0.39814
5	0.44022	0.48566	0.35976	0.51834	0.7319	0.76724	0.56242	0.53865
6	0.35708	0.37263	0.41224	0.42002	0.41782	0.44677	0.39543	0.39945
7	0.36274	0.3563	0.35705	0.37584	0.36126	0.35285	0.42889	0.37624
8	0.42479	0.4802	0.46969	0.48499	0.50624	0.61481	0.46588	0.48223
9	0.51656	0.52009	0.38655	0.4854	0.53795	0.58326	0.67759	0.5387
10	0.86924	1	0.39779	0.62284	0.8971	1	1	0.8279
11	0.36236	0.36278	0.51778	0.35366	0.3426	0.3517	0.37703	0.3761
12	0.42271	0.40969	0.47976	0.41952	0.35881	0.37121	0.41919	0.41423
13	0.51235	0.57073	0.35321	0.57143	0.96539	0.91068	0.67565	0.63189
14	0.679	0.71918	0.53881	0.67733	0.49505	0.57649	0.55174	0.61457
15	0.33333	0.33333	0.394	0.33691	0.33878	0.33333	0.35498	0.34499
16	0.68	0.68717	0.44649	0.56173	0.54193	0.62637	0.70884	0.62031
17	0.57416	0.58015	0.38234	0.52012	0.60446	0.65093	0.75804	0.59225
18	0.665	0.73714	0.42794	0.5546	0.64337	0.77105	0.75816	0.6535
19	0.48596	0.5089	0.33745	0.66027	0.88459	0.70522	0.61916	0.59669
20	0.37379	0.36719	0.35779	0.39093	0.37841	0.37238	0.45564	0.39165
21	0.49923	0.46134	0.41069	0.65187	0.37875	0.37984	0.43935	0.48115
22	0.3913	0.39773	0.88768	0.49217	0.33771	0.34641	0.33333	0.44031
23	0.34912	0.3472	0.45521	0.33333	0.33333	0.33397	0.36857	0.35718
24	0.54288	0.49394	0.40652	0.55827	0.41349	0.42465	0.5375	0.50159
25	0.7751	0.92355	0.54858	0.74867	0.56761	0.69427	0.59886	0.69793
26	0.48296	0.51817	0.78187	0.59249	0.40508	0.45609	0.40736	0.51268
27	0.43412	0.44337	0.33333	0.61617	0.67223	0.56884	0.55476	0.52279
28	0.38913	0.42092	0.35733	0.49899	0.62027	0.64687	0.47267	0.4767
29	0.38095	0.37347	0.34571	0.43078	0.3965	0.38218	0.4687	0.40587
30	0.60362	0.55029	0.44375	0.67293	0.41893	0.43809	0.50234	0.54088
31	0.43865	0.4488	0.33547	0.59119	0.67449	0.58329	0.57057	0.52429
32	0.64109	0.72189	0.3875	0.55999	0.8005	0.89851	0.81353	0.68376
33	1	0.89386	0.4516	1	0.53572	0.5764	0.64021	0.7738
34	0.38001	0.37181	0.45713	0.36548	0.34127	0.34546	0.39487	0.37964
35	0.54388	0.60174	0.55366	0.54245	0.48304	0.5813	0.52576	0.54439
36	0.43589	0.41872	0.35439	0.50002	0.43052	0.41849	0.53133	0.45566
37	0.6384	0.68241	0.47997	0.56093	0.53222	0.63441	0.63904	0.60057
38	0.81185	0.6904	0.4108	0.92165	0.48857	0.49789	0.61477	0.67842
39	0.42484	0.46503	0.33619	0.58981	1	0.8253	0.54218	0.57402
40	0.83256	0.81258	0.44429	0.63877	0.57245	0.65277	0.77019	0.69651
41	0.48404	0.54048	1	0.72146	0.40848	0.46175	0.38433	0.55904

42	0.54564	0.49456	0.40113	0.67598	0.40422	0.40635	0.49158	0.51327
43	0.36768	0.36107	0.382	0.3623	0.35508	0.35377	0.42515	0.37622
44	0.46661	0.44212	0.45759	0.49274	0.37047	0.38226	0.43457	0.44403
45	0.34974	0.34661	0.35941	0.35818	0.35676	0.34887	0.40709	0.36417
46	0.3708	0.36691	0.46776	0.35616	0.34407	0.35074	0.39327	0.37699
47	0.65414	0.5828	0.42386	0.90286	0.42878	0.43751	0.49896	0.59848
48	0.47573	0.48879	0.37259	0.48067	0.56077	0.59625	0.63743	0.52061
49	0.36745	0.3825	0.5097	0.4011	0.38504	0.41823	0.39194	0.40165
50	0.35953	0.35397	0.34802	0.38561	0.36665	0.35439	0.42806	0.37694

Trial 02:

weights = [0.4 0.1 0.1 0.1 0.1 0.1 0.1];

No	MRR	T_Tool	H	P	F_x	F_Y	R_a	GRG
1	0.36567	0.37173	0.57654	0.37136	0.35266	0.37041	0.37692	0.38823
2	0.66487	0.60996	0.42803	0.57861	0.47955	0.51951	0.65837	0.59335
3	0.37212	0.40552	0.36552	0.50865	0.59207	0.63503	0.42677	0.4422
4	0.38033	0.38161	0.41779	0.38043	0.38117	0.39768	0.44083	0.39208
5	0.44022	0.48566	0.35976	0.51834	0.7319	0.76724	0.56242	0.51862
6	0.35708	0.37263	0.41224	0.42002	0.41782	0.44677	0.39543	0.38932
7	0.36274	0.3563	0.35705	0.37584	0.36126	0.35285	0.42889	0.36832
8	0.42479	0.4802	0.46969	0.48499	0.50624	0.61481	0.46588	0.4721
9	0.51656	0.52009	0.38655	0.4854	0.53795	0.58326	0.67759	0.52571
10	0.86924	1	0.39779	0.62284	0.8971	1	1	0.83947
11	0.36236	0.36278	0.51778	0.35366	0.3426	0.3517	0.37703	0.3755
12	0.42271	0.40969	0.47976	0.41952	0.35881	0.37121	0.41919	0.4149
13	0.51235	0.57073	0.35321	0.57143	0.96539	0.91068	0.67565	0.60965
14	0.679	0.71918	0.53881	0.67733	0.49505	0.57649	0.55174	0.62746
15	0.33333	0.33333	0.394	0.33691	0.33878	0.33333	0.35498	0.34247
16	0.68	0.68717	0.44649	0.56173	0.54193	0.62637	0.70884	0.62925
17	0.57416	0.58015	0.38234	0.52012	0.60446	0.65093	0.75804	0.57927
18	0.665	0.73714	0.42794	0.5546	0.64337	0.77105	0.75816	0.65523
19	0.48596	0.5089	0.33745	0.66027	0.88459	0.70522	0.61916	0.56594
20	0.37379	0.36719	0.35779	0.39093	0.37841	0.37238	0.45564	0.38175
21	0.49923	0.46134	0.41069	0.65187	0.37875	0.37984	0.43935	0.47188
22	0.3913	0.39773	0.88768	0.49217	0.33771	0.34641	0.33333	0.43602
23	0.34912	0.3472	0.45521	0.33333	0.33333	0.33397	0.36857	0.35681
24	0.54288	0.49394	0.40652	0.55827	0.41349	0.42465	0.5375	0.50059
25	0.7751	0.92355	0.54858	0.74867	0.56761	0.69427	0.59886	0.71819
26	0.48296	0.51817	0.78187	0.59249	0.40508	0.45609	0.40736	0.50929
27	0.43412	0.44337	0.33333	0.61617	0.67223	0.56884	0.55476	0.49252
28	0.38913	0.42092	0.35733	0.49899	0.62027	0.64687	0.47267	0.45736
29	0.38095	0.37347	0.34571	0.43078	0.3965	0.38218	0.4687	0.39211
30	0.60362	0.55029	0.44375	0.67293	0.41893	0.43809	0.50234	0.54408
31	0.43865	0.4488	0.33547	0.59119	0.67449	0.58329	0.57057	0.49584
32	0.64109	0.72189	0.3875	0.55999	0.8005	0.89851	0.81353	0.67463
33	1	0.89386	0.4516	1	0.53572	0.5764	0.64021	0.80978
34	0.38001	0.37181	0.45713	0.36548	0.34127	0.34546	0.39487	0.37961
35	0.54388	0.60174	0.55366	0.54245	0.48304	0.5813	0.52576	0.54635
36	0.43589	0.41872	0.35439	0.50002	0.43052	0.41849	0.53133	0.43971
37	0.6384	0.68241	0.47997	0.56093	0.53222	0.63441	0.63904	0.60826
38	0.81185	0.6904	0.4108	0.92165	0.48857	0.49789	0.61477	0.68715
39	0.42484	0.46503	0.33619	0.58981	1	0.8253	0.54218	0.54579
40	0.83256	0.81258	0.44429	0.63877	0.57245	0.65277	0.77019	0.72213
41	0.48404	0.54048	1	0.72146	0.40848	0.46175	0.38433	0.54527
42	0.54564	0.49456	0.40113	0.67598	0.40422	0.40635	0.49158	0.50564
43	0.36768	0.36107	0.382	0.3623	0.35508	0.35377	0.42515	0.37101

44	0.46661	0.44212	0.45759	0.49274	0.37047	0.38226	0.43457	0.44462
45	0.34974	0.34661	0.35941	0.35818	0.35676	0.34887	0.40709	0.35759
46	0.3708	0.36691	0.46776	0.35616	0.34407	0.35074	0.39327	0.37621
47	0.65414	0.5828	0.42386	0.90286	0.42878	0.43751	0.49896	0.58913
48	0.47573	0.48879	0.37259	0.48067	0.56077	0.59625	0.63743	0.50394
49	0.36745	0.3825	0.5097	0.4011	0.38504	0.41823	0.39194	0.39583
50	0.35953	0.35397	0.34802	0.38561	0.36665	0.35439	0.42806	0.36748

Trial 03:
weights = [0.1 0.1 0.1 0.1 0.1 0.1 0.4];

No	MRR	T_Tool	H	P	F_x	F_Y	R_a	GRG
1	0.36567	0.37173	0.57654	0.37136	0.35266	0.37041	0.37692	0.3916
2	0.66487	0.60996	0.42803	0.57861	0.47955	0.51951	0.65837	0.5914
3	0.37212	0.40552	0.36552	0.50865	0.59207	0.63503	0.42677	0.4586
4	0.38033	0.38161	0.41779	0.38043	0.38117	0.39768	0.44083	0.41023
5	0.44022	0.48566	0.35976	0.51834	0.7319	0.76724	0.56242	0.55528
6	0.35708	0.37263	0.41224	0.42002	0.41782	0.44677	0.39543	0.40083
7	0.36274	0.3563	0.35705	0.37584	0.36126	0.35285	0.42889	0.38816
8	0.42479	0.4802	0.46969	0.48499	0.50624	0.61481	0.46588	0.48442
9	0.51656	0.52009	0.38655	0.4854	0.53795	0.58326	0.67759	0.57402
10	0.86924	1	0.39779	0.62284	0.8971	1	1	0.8787
11	0.36236	0.36278	0.51778	0.35366	0.3426	0.3517	0.37703	0.3799
12	0.42271	0.40969	0.47976	0.41952	0.35881	0.37121	0.41919	0.41385
13	0.51235	0.57073	0.35321	0.57143	0.96539	0.91068	0.67565	0.65864
14	0.679	0.71918	0.53881	0.67733	0.49505	0.57649	0.55174	0.58928
15	0.33333	0.33333	0.394	0.33691	0.33878	0.33333	0.35498	0.34896
16	0.68	0.68717	0.44649	0.56173	0.54193	0.62637	0.70884	0.63791
17	0.57416	0.58015	0.38234	0.52012	0.60446	0.65093	0.75804	0.63443
18	0.665	0.73714	0.42794	0.5546	0.64337	0.77105	0.75816	0.68317
19	0.48596	0.5089	0.33745	0.66027	0.88459	0.70522	0.61916	0.6059
20	0.37379	0.36719	0.35779	0.39093	0.37841	0.37238	0.45564	0.40631
21	0.49923	0.46134	0.41069	0.65187	0.37875	0.37984	0.43935	0.45391
22	0.3913	0.39773	0.88768	0.49217	0.33771	0.34641	0.33333	0.41863
23	0.34912	0.3472	0.45521	0.33333	0.33333	0.33397	0.36857	0.36265
24	0.54288	0.49394	0.40652	0.55827	0.41349	0.42465	0.5375	0.49897
25	0.7751	0.92355	0.54858	0.74867	0.56761	0.69427	0.59886	0.66532
26	0.48296	0.51817	0.78187	0.59249	0.40508	0.45609	0.40736	0.48661
27	0.43412	0.44337	0.33333	0.61617	0.67223	0.56884	0.55476	0.52871
28	0.38913	0.42092	0.35733	0.49899	0.62027	0.64687	0.47267	0.48242
29	0.38095	0.37347	0.34571	0.43078	0.3965	0.38218	0.4687	0.41844
30	0.60362	0.55029	0.44375	0.67293	0.41893	0.43809	0.50234	0.5137
31	0.43865	0.4488	0.33547	0.59119	0.67449	0.58329	0.57057	0.53542
32	0.64109	0.72189	0.3875	0.55999	0.8005	0.89851	0.81353	0.72636
33	1	0.89386	0.4516	1	0.53572	0.5764	0.64021	0.70184
34	0.38001	0.37181	0.45713	0.36548	0.34127	0.34546	0.39487	0.38406
35	0.54388	0.60174	0.55366	0.54245	0.48304	0.5813	0.52576	0.54091
36	0.43589	0.41872	0.35439	0.50002	0.43052	0.41849	0.53133	0.46834
37	0.6384	0.68241	0.47997	0.56093	0.53222	0.63441	0.63904	0.60845
38	0.81185	0.6904	0.4108	0.92165	0.48857	0.49789	0.61477	0.62802
39	0.42484	0.46503	0.33619	0.58981	1	0.8253	0.54218	0.58099
40	0.83256	0.81258	0.44429	0.63877	0.57245	0.65277	0.77019	0.70342
41	0.48404	0.54048	1	0.72146	0.40848	0.46175	0.38433	0.51535
42	0.54564	0.49456	0.40113	0.67598	0.40422	0.40635	0.49158	0.48942
43	0.36768	0.36107	0.382	0.3623	0.35508	0.35377	0.42515	0.38825
44	0.46661	0.44212	0.45759	0.49274	0.37047	0.38226	0.43457	0.43501

45	0.34974	0.34661	0.35941	0.35818	0.35676	0.34887	0.40709	0.37479
46	0.3708	0.36691	0.46776	0.35616	0.34407	0.35074	0.39327	0.38295
47	0.65414	0.5828	0.42386	0.90286	0.42878	0.43751	0.49896	0.54258
48	0.47573	0.48879	0.37259	0.48067	0.56077	0.59625	0.63743	0.55245
49	0.36745	0.3825	0.5097	0.4011	0.38504	0.41823	0.39194	0.40318
50	0.35953	0.35397	0.34802	0.38561	0.36665	0.35439	0.42806	0.38804

Trial 04:
weights = [0.2 0.1 0.1 0.1 0.1 0.1 0.3];

No	MRR	T_Tool	H	P	F_x	F_Y	R_a	GRG
1	0.36567	0.37173	0.57654	0.37136	0.35266	0.37041	0.37692	0.39048
2	0.66487	0.60996	0.42803	0.57861	0.47955	0.51951	0.65837	0.59205
3	0.37212	0.40552	0.36552	0.50865	0.59207	0.63503	0.42677	0.45313
4	0.38033	0.38161	0.41779	0.38043	0.38117	0.39768	0.44083	0.40418
5	0.44022	0.48566	0.35976	0.51834	0.7319	0.76724	0.56242	0.54306
6	0.35708	0.37263	0.41224	0.42002	0.41782	0.44677	0.39543	0.39699
7	0.36274	0.3563	0.35705	0.37584	0.36126	0.35285	0.42889	0.38155
8	0.42479	0.4802	0.46969	0.48499	0.50624	0.61481	0.46588	0.48032
9	0.51656	0.52009	0.38655	0.4854	0.53795	0.58326	0.67759	0.55791
10	0.86924	1	0.39779	0.62284	0.8971	1	1	0.86562
11	0.36236	0.36278	0.51778	0.35366	0.3426	0.3517	0.37703	0.37843
12	0.42271	0.40969	0.47976	0.41952	0.35881	0.37121	0.41919	0.4142
13	0.51235	0.57073	0.35321	0.57143	0.96539	0.91068	0.67565	0.64231
14	0.679	0.71918	0.53881	0.67733	0.49505	0.57649	0.55174	0.60201
15	0.33333	0.33333	0.394	0.33691	0.33878	0.33333	0.35498	0.3468
16	0.68	0.68717	0.44649	0.56173	0.54193	0.62637	0.70884	0.63502
17	0.57416	0.58015	0.38234	0.52012	0.60446	0.65093	0.75804	0.61604
18	0.665	0.73714	0.42794	0.5546	0.64337	0.77105	0.75816	0.67386
19	0.48596	0.5089	0.33745	0.66027	0.88459	0.70522	0.61916	0.59258
20	0.37379	0.36719	0.35779	0.39093	0.37841	0.37238	0.45564	0.39812
21	0.49923	0.46134	0.41069	0.65187	0.37875	0.37984	0.43935	0.4599
22	0.3913	0.39773	0.88768	0.49217	0.33771	0.34641	0.33333	0.42443
23	0.34912	0.3472	0.45521	0.33333	0.33333	0.33397	0.36857	0.3607
24	0.54288	0.49394	0.40652	0.55827	0.41349	0.42465	0.5375	0.49951
25	0.7751	0.92355	0.54858	0.74867	0.56761	0.69427	0.59886	0.68295
26	0.48296	0.51817	0.78187	0.59249	0.40508	0.45609	0.40736	0.49417
27	0.43412	0.44337	0.33333	0.61617	0.67223	0.56884	0.55476	0.51665
28	0.38913	0.42092	0.35733	0.49899	0.62027	0.64687	0.47267	0.47407
29	0.38095	0.37347	0.34571	0.43078	0.3965	0.38218	0.4687	0.40966
30	0.60362	0.55029	0.44375	0.67293	0.41893	0.43809	0.50234	0.52383
31	0.43865	0.4488	0.33547	0.59119	0.67449	0.58329	0.57057	0.52223
32	0.64109	0.72189	0.3875	0.55999	0.8005	0.89851	0.81353	0.70912
33	1	0.89386	0.4516	1	0.53572	0.5764	0.64021	0.73782
34	0.38001	0.37181	0.45713	0.36548	0.34127	0.34546	0.39487	0.38258
35	0.54388	0.60174	0.55366	0.54245	0.48304	0.5813	0.52576	0.54272
36	0.43589	0.41872	0.35439	0.50002	0.43052	0.41849	0.53133	0.45879
37	0.6384	0.68241	0.47997	0.56093	0.53222	0.63441	0.63904	0.60839
38	0.81185	0.6904	0.4108	0.92165	0.48857	0.49789	0.61477	0.64773
39	0.42484	0.46503	0.33619	0.58981	1	0.8253	0.54218	0.56926
40	0.83256	0.81258	0.44429	0.63877	0.57245	0.65277	0.77019	0.70965
41	0.48404	0.54048	1	0.72146	0.40848	0.46175	0.38433	0.52533
42	0.54564	0.49456	0.40113	0.67598	0.40422	0.40635	0.49158	0.49483
43	0.36768	0.36107	0.382	0.3623	0.35508	0.35377	0.42515	0.3825
44	0.46661	0.44212	0.45759	0.49274	0.37047	0.38226	0.43457	0.43821

45	0.34974	0.34661	0.35941	0.35818	0.35676	0.34887	0.40709	0.36906
46	0.3708	0.36691	0.46776	0.35616	0.34407	0.35074	0.39327	0.38071
47	0.65414	0.5828	0.42386	0.90286	0.42878	0.43751	0.49896	0.55809
48	0.47573	0.48879	0.37259	0.48067	0.56077	0.59625	0.63743	0.53628
49	0.36745	0.3825	0.5097	0.4011	0.38504	0.41823	0.39194	0.40073
50	0.35953	0.35397	0.34802	0.38561	0.36665	0.35439	0.42806	0.38119

Trial 05:
weights = [0.3 0.1 0.1 0.1 0.1 0.1 0.2];

No	MRR	T_Tool	H	P	F_x	F_Y	R_a	GRG
1	0.36567	0.37173	0.57654	0.37136	0.35266	0.37041	0.37692	0.38935
2	0.66487	0.60996	0.42803	0.57861	0.47955	0.51951	0.65837	0.5927
3	0.37212	0.40552	0.36552	0.50865	0.59207	0.63503	0.42677	0.44767
4	0.38033	0.38161	0.41779	0.38043	0.38117	0.39768	0.44083	0.39813
5	0.44022	0.48566	0.35976	0.51834	0.7319	0.76724	0.56242	0.53084
6	0.35708	0.37263	0.41224	0.42002	0.41782	0.44677	0.39543	0.39316
7	0.36274	0.3563	0.35705	0.37584	0.36126	0.35285	0.42889	0.37493
8	0.42479	0.4802	0.46969	0.48499	0.50624	0.61481	0.46588	0.47621
9	0.51656	0.52009	0.38655	0.4854	0.53795	0.58326	0.67759	0.54181
10	0.86924	1	0.39779	0.62284	0.8971	1	1	0.85254
11	0.36236	0.36278	0.51778	0.35366	0.3426	0.3517	0.37703	0.37697
12	0.42271	0.40969	0.47976	0.41952	0.35881	0.37121	0.41919	0.41455
13	0.51235	0.57073	0.35321	0.57143	0.96539	0.91068	0.67565	0.62598
14	0.679	0.71918	0.53881	0.67733	0.49505	0.57649	0.55174	0.61473
15	0.33333	0.33333	0.394	0.33691	0.33878	0.33333	0.35498	0.34463
16	0.68	0.68717	0.44649	0.56173	0.54193	0.62637	0.70884	0.63214
17	0.57416	0.58015	0.38234	0.52012	0.60446	0.65093	0.75804	0.59766
18	0.665	0.73714	0.42794	0.5546	0.64337	0.77105	0.75816	0.66454
19	0.48596	0.5089	0.33745	0.66027	0.88459	0.70522	0.61916	0.57926
20	0.37379	0.36719	0.35779	0.39093	0.37841	0.37238	0.45564	0.38994
21	0.49923	0.46134	0.41069	0.65187	0.37875	0.37984	0.43935	0.46589
22	0.3913	0.39773	0.88768	0.49217	0.33771	0.34641	0.33333	0.43023
23	0.34912	0.3472	0.45521	0.33333	0.33333	0.33397	0.36857	0.35876
24	0.54288	0.49394	0.40652	0.55827	0.41349	0.42465	0.5375	0.50005
25	0.7751	0.92355	0.54858	0.74867	0.56761	0.69427	0.59886	0.70057
26	0.48296	0.51817	0.78187	0.59249	0.40508	0.45609	0.40736	0.50173
27	0.43412	0.44337	0.33333	0.61617	0.67223	0.56884	0.55476	0.50458
28	0.38913	0.42092	0.35733	0.49899	0.62027	0.64687	0.47267	0.46571
29	0.38095	0.37347	0.34571	0.43078	0.3965	0.38218	0.4687	0.40089
30	0.60362	0.55029	0.44375	0.67293	0.41893	0.43809	0.50234	0.53395
31	0.43865	0.4488	0.33547	0.59119	0.67449	0.58329	0.57057	0.50903
32	0.64109	0.72189	0.3875	0.55999	0.8005	0.89851	0.81353	0.69187
33	1	0.89386	0.4516	1	0.53572	0.5764	0.64021	0.7738
34	0.38001	0.37181	0.45713	0.36548	0.34127	0.34546	0.39487	0.38109
35	0.54388	0.60174	0.55366	0.54245	0.48304	0.5813	0.52576	0.54453
36	0.43589	0.41872	0.35439	0.50002	0.43052	0.41849	0.53133	0.44925
37	0.6384	0.68241	0.47997	0.56093	0.53222	0.63441	0.63904	0.60832
38	0.81185	0.6904	0.4108	0.92165	0.48857	0.49789	0.61477	0.66744
39	0.42484	0.46503	0.33619	0.58981	1	0.8253	0.54218	0.55752
40	0.83256	0.81258	0.44429	0.63877	0.57245	0.65277	0.77019	0.71589
41	0.48404	0.54048	1	0.72146	0.40848	0.46175	0.38433	0.5353
42	0.54564	0.49456	0.40113	0.67598	0.40422	0.40635	0.49158	0.50023
43	0.36768	0.36107	0.382	0.3623	0.35508	0.35377	0.42515	0.37676
44	0.46661	0.44212	0.45759	0.49274	0.37047	0.38226	0.43457	0.44142

45	0.34974	0.34661	0.35941	0.35818	0.35676	0.34887	0.40709	0.36332
46	0.3708	0.36691	0.46776	0.35616	0.34407	0.35074	0.39327	0.37846
47	0.65414	0.5828	0.42386	0.90286	0.42878	0.43751	0.49896	0.57361
48	0.47573	0.48879	0.37259	0.48067	0.56077	0.59625	0.63743	0.52011
49	0.36745	0.3825	0.5097	0.4011	0.38504	0.41823	0.39194	0.39828
50	0.35953	0.35397	0.34802	0.38561	0.36665	0.35439	0.42806	0.37434

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