# ISLAMIC UNIVERSITY OF TECHNOLOGY (IUT)

# Effect of Ball Burnishing Parameters on the Surface Roughness by a Burnishing Tool with Magnetic Ball Holding Device

**MSc Engineering (Mechanical) Thesis**

**by**


**Muhammad Khaled Aminul Hasan**

**Department of Mechanical and Chemical Engineering**

**Islamic University of Technology (IUT)**

**Gazipur-1704, Bangladesh.**

**Student No: 121608**


**Supervised by**

**Prof. Dr. Nurul Absar Chowdhury**

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## CERTIFICATION

The thesis titled **Effect of ball burnishing parameters on the surface roughness by a burnishing tool with magnetic ball holding device** Submitted by Muhammad Khaled Aminul Hasan St.No.121608 of Academic Year 2012-2013 has been found as satisfactory and accepted as partial fulfillment of the requirement for the Degree MSc. in Mechanical Engineering on 27 March,2017.

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It is hereby declared that the thesis, entitled **Effect of ball burnishing parameters on the surface roughness by a burnishing tool with magnetic ball holding device** is based on work contemplated by me and no part of it has been presented, previously published or written for the award of any Degree or Diploma. This research work was carried out in the Department of MCE ,IUT for the partial fulfillment of the requirements for the degree of MSc. in Mechanical Engineering.


**Dr. Nurul Absar Chowdhury**
Professor
Department of MCE
Islamic University of Technology

**Muhammad Khaled Aminul Hasan**
Student No: 121608
Academic Year: 2012-2013
Date:

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# ABSTRACT

In this study four burnishing parameters were selected for optimizing the burnishing process using Taguchi method. The examined burnishing parameters include: (1) Burnishing speed, (2) Force and (3) Feed rate and (4) Ball diameter. Other parameters such as number of burnishing passes, and penetration depth are considered constant in this study. For each parameter, 5(five) levels were considered. According to Taguchi method with 4 (four) independent parameters, 25 experiments are conducted. 5 (five) coded levels were used for each parameter and MINITAB software has used for data analysis. The purpose of this work is to study the relationships between surface finish and the ball burnishing process parameters. The paper deals with the effect of burnishing process on Mild Steel using Lathe. Surface roughness generated after the turning operation was used to ball burnishing. For better response parameter a ball burnishing tool is developed with magnetic ball holding device which reduces the friction between ball and bearing. Moreover, the tool is designed for using of different ball diameter. Experimental work was carried out on a Conventional lathe and surface roughness is determined. From all experimental data a mathematical model is established by Dimensional Analysis. The optimal burnishing parameters were found for the surface roughness by using of Taguchi technique with Lower is better S/N ratio and validation of the process by Response Surface Methodology (RSM) (systematic method for using the influential factors in a process for improvement and optimization). It was found that the optimal burnishing parameters for the best surface finish was at burnishing speed of 155 rpm, burnishing feed of 0.1 mm/rev, force 78 N and ball diameter 11 mm. The effects of burnishing parameters (i.e., burnishing speed, force, feed rate and ball diameter) on the surface roughness was investigated by analysis of variance (ANOVA). It was found that the burnishing feed and ball diameter has the most influential effect on the surface roughness, followed by the burnishing force, and least influence by the speed.

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| | | |
|---|---|---|
| ANOVA | : | Analysis of Variance |
| RSM | : | Response Surface Methodology |
| CNC | : | Computer Numerical Control |
| LPP | : | Linear Programming Problem |
| OA | : | Orthogonal Array |
| OR | : | Operation Research |
| SNR | : | Signal-to-noise ratio |
| ECL | : | Electrical Communication Laboratories |
| SST | : | Total sum of squares |
| SSE | : | Sum of squares of the residual error |
| SSR | : | Regression sum of squares |
| DOE | : | Design of Experiment |
| OD | : | Outside Diameter |
| Ra | : | Surface roughness |
| RPM | : | Revolution per Minute |
| $R^2$ | : | Coefficient of determination |
| ε | : | Relative depth of penetration |

# CHAPTER-1

# INTRODUCTION

## 1.1 Background

Surface quality is of immense significance in the performance of mechanical components. Despite the manufacturing process used, the the surface roughness of diverse asperities usually exists in almost all surfaces of mechanical parts such that obtained in machined casting dies or hot-rolled plates. As a result, more concentration is paid in the finishing process for the period of manufacturing. Methods that are commonly used to get the the better surface finish and produce low values of roughness include grinding, lapping, honing, and polishing. However, the more widely used method of surface finishing is burnishing. In this method, a large contact pressure is applied on the surface of the workpiece by a smooth roller (roller burnishing) or a ball (ball burnishing) to reason plastic deformation of surface irregularities. The high burnishing pressure, exceeding the yield strength, causes roughness crests to flow toward the valleys and thus coats all the texture of the rough surface, resulting in smoother surfaces. This method of cold-working surface treatment is different from other surface treatments, such as shot peening and sand blasting in such a way it creates a good surface finish, boost up dimensional and shape accuracy, enhances surface hardness, and also induces residual compressive stresses at the metallic surface layers. Several works have examined the result of burnishing on improving mechanical properties, and shown that proper design of burnishing process can lead to increased hardness [1], to enhance quality of surface finish, to increase utmost residual stress in compression, to prevent corrosion and stress corrosion cracking, and to enhance the wear resistance and fatigue life of the workpiece [2]. In general, burnishing leads to changes in the microstructure of the burnished surface. However, excessive burnishing can lead to subsurface cracks which cause spall, i.e., a phenomenon where the upper layer of a surface flakes off of the bulk material. There are several controlling parameters that can have an effect on the workpiece surface properties. These parameters include: burnishing speed, feed rate, force (or pressure), the number of burnishing passes, workpiece material, ball material, ball size, and lubricant. In general, the two most commonly cited parameters affecting surface finish are the burnishing force and the feed rate. Despite a large number of works on burnishing of round workpieces such as crankshafts and bearing races, the treatment of cylindrical surfaces by either roller or ball burnishing is yet to be entirely investigated [3]. Considering the above, this work examines the use of a newly developed ball burnishing tool to give enhanced surface properties for Mild steel. The tool was specifically designed to treat cylindrical surfaces in a rational experimental time. In order to explore the optimum combination of burnishing parameters, several experiments were designed and performed on a machining center based on the Taguchi method of optimization with the Taguchi L25 Matrix and validation of the process by Response Surface Methodology (RSM) (systematic method for using the influential factors in a process for improvement and optimization). The effects of burnishing parameters (i.e., burnishing speed, force, feed rate and ball diameter) on the surface roughness were investigated by analysis of variance (ANOVA), as presented by the mean surface roughness (Ra).

### 1.2 Objectives with specific aims

a) To design a flexible ball burnishing tool with a possibility of use different ball diameter
b) To explore the optimum combination of burnishing parameter with Taguchi L25 matrix for surface roughness of Mild steel specimen
c) To find out the effect of burnishing speed, force, feed and ball diameter on surface roughness of mild steel for the designed tool
d) To develop an analytical method for optimization of burnishing parameter for surface roughness

### 1.3 Expected outcomes

a) An effective surface finishing tool for Mild steel and other materials.
b) The empirical formula for determination of surface roughness for given burnishing conditions.
c) Mathematical model giving relationship between output parameter and input parameter of burnishing process.
d) Analytical model to determine optimum condition of burnishing parameter.

### 1.4 Burnishing

Burnishing is the plastic deformation of a surface due to sliding contact with another entity. Visually, burnishing marks the texture of a rough surface and makes it shinier. Burnishing may arise on any sliding surface if the contact stress locally exceeds the yield strength of the material.

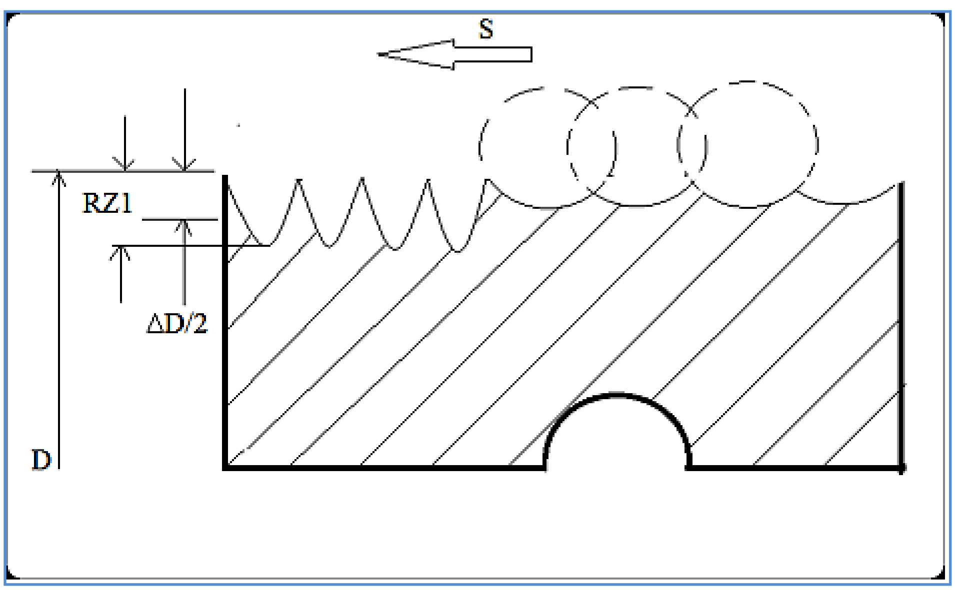


Fig. 1.1 Deformation of the surface in ball burnishing

Classification of burnishing process:

A) Based on motion of the tool on the surface.

i. Normal or ordinary burnishing

ii. Vibratory burnishing

B) Based on the shape of deforming element

i. Ball burnishing

ii. Roller burnishing

C) Based on the application of deforming element

i. Rigid

ii. Flexible

### 1.5 Comparison between Ball and Roller Burnishing

| SL | Ball Burnishing | Roller Burnishing |
|---|---|---|
| 1 | The deforming element is a hard steel ball. | The deforming element is a hard steel roller. |
| 2 | Point contact and rolling friction between ball and workpiece. | Line contact and sliding friction between roller and workpiece. |
| 3 | Deformation is localized in a zone adjacent to the ball. | More chances of deformation of the entire blank compared to the ball burnishing. |
| 4 | For the same radial force, gives high specific pressure, better surface finish, more fatigue strength, microhardness and depth of work hardened layer. | It gives less specific pressure, poor surface, lower fatigue strength, microhardness and depth of work hardened layer. |
| 5 | Low production rate | High production rate |

## 1.6 Mechanics of Burnishing

To know about burnishing, let us first look at the simple case of a hardened ball on a workpiece. If the ball is pressed directly into the surface of that workpiece, stresses develop in both objects around the area where they contact. As this normal force increases, both the ball and the surface of the workpiece are deformed. The deformation caused by the hardened ball is different depending on the level of the force pressing against it. If the force on it is small, when the force is released both the ball and the surface of the workpiece will return to their original, unreformed shape. In this case, the stresses in the plate are always less than the yield strength of the material, so the deformation is purely elastic. Since it was given that the workpiece is softer than the ball, the plate's surface will always deform more. (This is not necessarily true. For instance: if both items are steel, hardened steel has the same Young's Modulus as soft steel.) If a larger force is used, there will also be plastic deformation and the workpiece surface will be permanently altered. (In this situation, hardness does play a role, as increasing hardness will delay plastic deformation.) A bowl-shaped indentation will be left behind, surrounded by a ring of raised material that was displaced by the ball. The stresses between the ball and workpiece are described in more detail by Hertzian stress theory.

Now consider what happens if the external force on the ball drags it across the workpiece. In this case, the force on the ball can be decomposed into two component forces: one normal to the workpiece surface, pressing it in, and the other tangential, dragging it along. As the tangential component is increased, the ball will start to slide along the workpiece. At the same time, the normal force will deform both objects, just as with the static situation. If the normal force is low, the ball will rub against the workpiece but not permanently alter its surface. The rubbing action will create friction and heat, but it will not leave a mark on the plate. However, as the normal force increases, eventually, the stresses in the workpiece surface will exceed its yield strength. When this happens the ball will plow through the surface and create a trough behind it. The plowing action of the ball is burnishing. Burnishing also occurs when the ball can rotate, as would happen in the above scenario if another workpiece was brought down from above to induce downwards loading, and at the same time to cause rotation and translation of the ball, or in the case of a ball bearing.

Burnishing also occurs on surfaces that conform to each other, such as between two flat plates, but it happens on a microscopic scale. Even the smoothest of surfaces will have imperfections if viewed at a high enough magnification. The imperfections that extend above the general form of a surface are called asperities, and they can plow material on another surface just like the ball dragging along the plate. The combined effect of many of these asperities produces the smeared texture that is associated with burnishing.

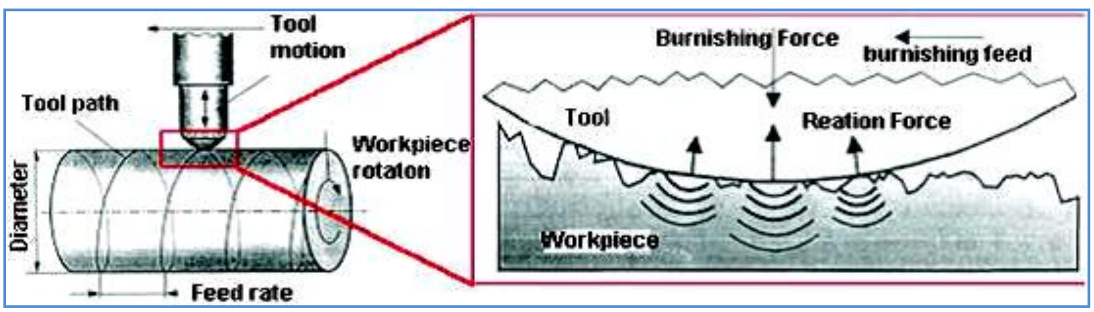


Fig 1.2 Mechanics of Burnishing

### 1.7 Burnishing in manufacturing

Burnishing processes are used in manufacturing to improve the size, shape, surface finish, or surface hardness of a workpiece. It is essentially a forming operation that occurs on a small scale. The benefits of burnishing often include: Combats fatigue failure, prevents corrosion and stress corrosion, textures surfaces to eliminate visual defects, closes porosity, creates surface compressive residual stress.

There are several forms of burnishing processes, the most common are roller burnishing and ball burnishing (a subset of which is also referred to as ballizing). In both cases, a burnishing tool runs against the workpiece and plastically deforms its surface. In some instances of the latter case (and always in ballizing), it rubs, in the former, it generally rotates and rolls. The workpiece may be at ambient temperature or heated to reduce the forces and wear on the tool. The tool is usually hardened and coated with special materials to increase its life. Ball burnishing, or ballizing, is a replacement for other bore finishing operations such as grinding, honing, or polishing. A ballizing tool consists of one or more over-sized balls that are pushed through a hole. The tool is similar to a broach, but instead of cutting away material, it plows it out of the way. Ball burnishing is also used as a deburring operation. It is especially useful for removing the burr in the middle of a through hole that was drilled from both sides.

Ball burnishing tools of another type are sometimes used in CNC milling centres to follow a ball-nosed milling operation: the hardened ball is applied along a zig-zag tool path in a holder similar to a ball-point pen, except that the 'ink' is pressurized, recycled lubricant. This combines the productivity of a machined finish which is achieved by a 'semi-finishing' cut, with a better finish than obtainable with slow and time-consuming finish cuts. The feed rate for burnishing is that associated with 'rapid traverse' rather than finish machining. Roller burnishing, or surface rolling, is used on cylindrical, conical, or disk-shaped workpieces. The tool resembles a roller bearing, but the rollers are generally very slightly tapered so that their envelope diameter can be accurately adjusted. The rollers typically rotate within a cage, as in a roller bearing. Typical applications for roller burnishing include hydraulic system components, shaft fillets, and sealing surfaces. Very close control of size can be exercised. Burnishing also occurs to some extent in machining processes. In turning, burnishing occurs if the cutting tool is not sharp, if a large negative rake angle is used, if a very small depth of cut is used, or if the workpiece material is gummy. As a cutting tool wears, it becomes blunter and the burnishing effect becomes more pronounced. In grinding, since the abrasive grains are randomly oriented and some are not sharp, there is always some amount of burnishing. This is one reason the grinding is less efficient and generates more heat than turning. In drilling, burnishing occurs with drills that have lands to burnish the material as it drills into it. Regular twist drills or straight fluted drills have 2 lands to guide them through the hole. On burnishing drills, there are 4 or more lands, similar to reamers.

### 1.8 Applications

Burnishing tools are being used in sectors like

- Automobile
- Aircraft
- Defense, Spacecraft, Railways
- Textile, Machine Tool, Motors and Pump Industry
- Hydraulic and Pneumatic Farm Equipment
- Home Appliances
- Areas where close tolerance and superior surface finish is required

### 1.9 Benefits

- Short cycle time and elimination of setting up and auxiliary processing time.
- For use with either conventional or CNC controlled machines.
- Complete processing in one setting.
- Removes no material and generates no waste
- Easily reproducible
- Low lubricant requirements.
- Low noise emission.
- Long tool life.
- No dimensional change through tool wear.

# CHAPTER-2

## LITERATURE REVIEW

### 2.1 Burnishing effects on the surface roughness of the different workpiece

Khalid. S. Rababa et al. (2011) studied the outcome of roller burnishing on mechanical performance and surface quality of Alloy steel. Based on their study it was observed that the stress of material has been increased of about 150 MPa, Roller Burnishing has a positive effect on the surface roughness of alloy steel. The surface roughness decreased with increasing burnishing force. The enhancement percentage on the surface quality was 12.5%, Roller Burnishing has an effect on the ultimate tensile strength, the UTS has been increased by 166 MPa, Roller Burnishing has an effect on ductility of material; the percentage elongation of material has been increased of 13.6% RB has an effect on cross -sectional area, the reduction of cross-sectional area has been increased by 1.8 % [4].

W Bouzid Sai et al. (2005) worked on Finite element modeling of burnishing of AISI 1042 steel. Based on their work it was predicted that Good correlation was found based on roughness experimental results. For the range of feed, roughness results agree well qualitatively and quantitatively with results found by using Hertz contact theory [5].

L.N. Lopez De Lacalle et al. (2007) worked on the effect of ball burnishing on heat treated steel and inconel 718 milled surface. Based on their work it was observed that the hydrostatic ball burnishing process is a relatively new enhanced surface treatment on free-form parts, previously obtained by milling. It is observed that maximum pressure 20 MPa leads to the highest quality improvement in the steels, but not in the Inconel case.

In this ductile material high pressures induce the appearance of grooves on burnished surface. The best results are with pressures from 10 to 15 MPa [6]. It was seen that the surface roughness reduces after burnishing process.

Ugur Esme et al. (2010) worked on Use of gray based Taguchi method in ball Burnishing process for the optimization of Surface roughness and microhardness of AA 7075 aluminum alloy. Based on their work it was predicted that the burnishing force has a maximum contribution of affecting the surface roughness. The contribution of burnishing force and no. of tool passes is more which 71.59% for force and 15.75% for no.of passes [7]. N.S.M. El Tayeb et al. (2007) worked on Influence of roller burnishing contact width and burnishing orientation on surface quality and triblogical behavior of Aluminum 6061 Based on their work it was predicted that the Optimum ranges of burnishing speed and force are identified to be 250–420 rpm for 1mm roller contact width. Burnishing force above 220N is capable of decreasing the surface roughness by 35%. Below this limit, the surface roughness starts to deteriorate plastically; Burnishing with smaller roller contact width (1 mm) is capable of improving the surface roughness up to 40%. Burnishing speed 110 rpm yields the highest Improvement in hardness, as much as 30% increase. Increasing burnishing force has a negative impact on the wear resistance of burnished Aluminum 6061 surfaces [8]. Aysus Sagbas et al. (2011) worked on Analysis and optimization of surface roughness in the ball burnishing process using response surface methodology and desirability function. Based on his work it was predicted that the surface roughness reduces with increasing burnishing force and no. of tool passes [9]. N.S.M. EL-Tayeb et al. (2011) worked on Prediction of burnishing surface integrity using Radial Basis Function. Based on their work a radial basic function algorithm was successfully used to predict the surface roughness for burnishing brass surface [10]. Highest reduction in Ra (200%) was achieved at lower burnishing speeds. Increasing both burnishing speed and depth showed a negative impact on the improvement of surface roughness especially at higher feed rate. 50 % improvement in the hardness of burnishing surfaces was achieved at lower feed rate.

It is seen that from the above literature review ball diameter is not considered as burnishing parameter, in this study ball diameter is regarded as burnishing parameter and find out the effect of different ball diameter on surface roughness.

**2.2 Necessity of design a ball burnishing Tool**

G. Schneider Jr et al. (2002) It is pragmatic that the conventional machining methods such as turning and milling leave inherent irregularities on machined surfaces and it becomes necessary to very often resort to a series of finishing operations with high costs [11]. N.S.M. El Tayeb et al. (2009) However, conventional finishing processes like grinding, honing and lapping are traditionally used finishing processes, but these methods essentially depend on chip removal to attain the desired surface finish, these machining chips may cause further surface abrasion and geometrical tolerance problems. Accordingly, burnishing process offers an attractive post-machining alternative due to it's chipless and relatively simple operations [12]. Many researchers have done their works by developing different types of burnishing tools i.e. ball and roller burnishing and making them ready to use with conventional machine tools viz. Lathe and milling.

A.M. Hassan et al. (2000) Developed ball burnishing tool with different ball diameters and examined the effects of parameters on non-ferrous workpiece materials like machining brass and Al-Cu alloy [13]. Mieczyslaw Korzynski et al. (2010) Developed the centreless burnishing device to conduct burnishing process on long length workpieces smoothly [14]. Effect of roller burnishing tool width and burnishing orientation was studied by N.S.M. El-Tayeb et al. (2011) to find the effect of different parameters on tribological properties as well as on surface qualities [10]. Fang-Jung Shiou et al. (2010) Sliding contact with rolling contact type burnishing tool is developed and effect of burnishing force is investigated on surface roughness on PDS5 plastic injection mold steel [15]. L.N. Lo´pez de Lacalle et al. (2007) Proposed reduction in lead time together with production cost in ball burnishing process carried out on milling centre, using a large radial depth of cut in the previous ball end milling operation, together with a small radial depth during burnishing produces acceptable surface roughness [6]. H. Hamadache et al. (2006) Studied the Characteristics of Rb40 steel superficial layer properties under ball and roller burnishing and concluded that roller burnishing will give optimum surface roughness while ball burnishing becomes effective in case of hardness [16]. P. N. Patel et al.( 2014) ,These study deals with optimization of newly design ball burnishing tool is used carried out experiment on conventional lathe machine with burnishing process parameters using taguchi analysis method. The work piece and ball materials used is Aluminum Alloy 6061 and high chromium high carbon with 8mm diameter. The levels of input process parameters are selected on basis of one factor at a time experiment are burnishing force, burnishing feed, burnishing speed and number of passes. The response parameters are hardness. The optimal parameters for hardness are as follows: burnishing speed 250 RPM, burnishing feed rate 0.06 mm/rev, burnishing force, 8 Kgf, No. of passes 5 [17]. Anil Jetani et al. (2015) burnishing is a cool working process in which plastic miss happening happens by applying a weight through a hard and smooth ball or roller on metallic surfaces. It is a finishing and strengthening methodology. Improvements in surface finish and surface hardness is genuine concern in organizations for achieving distinct advantage. Roller cleaning is a frigid working technique which conveys a fine surface wrap up by the planetary upset of hardened disturbs more than a depleted or turned metal surface. The nature of burnishing machined parts is altogether influenced by different parameters utilized as a part of the procedure. The point of present work is to study the four parameters of the roller burnishing process, such as number of passes, force, feed rate, and burnishing speed. Their impact on two reactions such as surface hardness and surface roughness of the test examples may contemplate. Outline of examinations are utilized to inspect the relationship between one or more reaction variables and an arrangement of quantitative exploratory variables or components. These systems are frequently utilized after recognized the critical controllable variables and to discover the element choice that enhances the reaction [18]. Pavan Kumar et al. (2013), Today's metal processing industries are often interested to induce compressive residual stresses in the several components which they will come across in fabrication processes daily. The conventional methods of finishing process viz. grinding, broaching used to improve the surface finish of the metallic components, but the burnishing process which is having same role to play in finishing process has many advantages associated with it fulfilling above said requirement successfully. This paper presents results of the study about design and developmental issues of Ball burnishing tool. This tool is used to perform burnishing process successfully by controlling different parameters [19]. Vipul Patel et al. (2015), Burnishing is a chip less finishing method, which employs a rolling tool, pressed against the work piece, in order to achieve plastic deformation of the surface layer. Burnishing processes are largely considered in industrial cases in order to restructure surface characteristics. These study deals with investigating the effect of burnishing process parameter with roller burnishing tool on CNC Machine using Response Surface Methodology and develop the Mathematical Model. The Work piece material and Tool material are Aluminium Alloy 6351 T6 and Roller of Carbide used as burnishing tool. As per previous research the effect of burnishing speed, feed, ball diameter, burnishing force and no. of tool passes playing important role on the quality of the work surface produced and its wearing characteristics. The process parameters used are cutting speed, interference, tool feed, number of tool passes and response Parameters are Surface Roughness and Hardness. In design of experiment total L31 experiment has been carried out with four factors and five levels. From the experiment it was identified that the minimum Surface Roughness obtained is 0.080 μm at 450 rpm, 0.064 mm/rev, 2 mm, 4 for Cutting Speed, Feed Rate, Interference and Number of Tool Passes respectively. It was identified that the maximum Hardness obtained is 107 BHN at 450 rpm, 0.064 mm/rev, 5 mm, 3 for Cutting Speed, Feed Rate, Interference and Number of Tool Passes respectively. The analysis of variance (ANOVA) was performed to statistically analyze the results [20]. S. H. Tang et al.

(2012), Artificial Neural Network (ANN) approach is a fascinating mathematical tool, which can be used to simulate a wide variety of complex scientific and engineering problems. Due to its highly reliable prediction quality, the usage of it is growing rigorously and had already become an ultimate tool for various applications in the field of engineering. In this study an ANN technique was used to predict friction coefficient of roller burnishing AL6061 for two orientations which is parallel burnishing orientation (PB) and cross burnishing orientation (CB). The input parameters were defined by widths of roller curvature (7.5mm, 8mm and 8.5mm), burnishing speeds (110rpm, 230rpm, 330rpm and 490rpm), and burnishing forces (155.06N, 197.45N, 239.83N and 282.22N) while the output parameter was friction coefficient. 173 data was used for training the ANN and another 115 data was used to test the ANN. 60 different configurations of ANN was trained by using 6 different training algorithms. It was found that feed-forward back-propagation network with 15 neurons in hidden layer that was trained by Levenberg-Marquardt training algorithm gave the best result when compared to other training algorithms used. From the results it was found that the training performance and prediction performance was 0.000809 and 0.710 respectively. From this study, it became obvious that the selected ANN with the configuration and training algorithm proved to be the most suitable [21]. Deepak Mahajan et al. (2013), Burnishing is a very simple and effective method for improvement in surface finish and can be carried out using existing machines, such as lathe. On account of its high productivity, it also saves more on production costs than other conventional processes such as super finishing, honing and grinding. Moreover, the burnished surface has a high wear resistance and better fatigue life. A literature survey being specifically focused on Ball burnishing process is done .It gives a thorough idea about various workpiece materials, various cutting tools and machine tools, process parameters ,lubricants, variable measured and methodology used as well as the prominent levels of each, being observed in the researches till today [22].

It is seen that the tools that are now used to burnishing with balls of different diameter fails to remove or reduce the friction in axial direction which is vital for forming the surface roughness and surface hardness. Besides, the mechanism used for holding the balls is complicated and costly and may sometimes added additional friction. So, it is required to develop a tool with attachment where the ball can rotate without friction in both direction and have a simple ball holding device.

### 2.3 Flexible magnetic holding ball burnishing Tool

In this study, a flexible ball burnishing tool is designed for carrying out the experiments. The hardness of material is not uniform, so the flexible tool is required to harmonize the different hardness of different point of materials. Moreover to reduce the friction between workpiece and ball three bearing is used which is rolled with the ball. For using various diameter ball a small chuck is used and by magnetizing the chuck the ball is held on the bearing. Since the tool reduces the friction, as a result better surface finish is achieved. Furthermore, the opportunity to use various ball diameter in one tool. Previously designed tool has no flexibility to change the ball simply. It requires modifying the casing of the tool which is a tedious job. Considering of this issue, in this study, a ball burnishing tool is designed with a magnetic ball holding device.

### 2.4 Methods Used in for optimization of burnishing parameters

There are a number of methods used to predict optimized values of burnishing process. Many researchers studied and worked on the optimization methods. Fang-Jung Shiou et al. (2003) used Taguchi L18 Orthogonal array technique and ANOVA to investigate the surface roughness value. Best on their results The Vickers hardness scale of the tested specimen was improved from about 338 to 480 after ball burnishing process. The hardened layer thickness was about 30 μm. By applying the optimal burnishing parameters for plane burnishing to the surface finish of the freeform surface mold cavity, the surface roughness improvement of the injection part of the plane surface was about 62.9% and that on the freeform surface was about 77.8% [23]. U.M. Shirsat et al. (2004) studied the parametric analysis of combined turning and ball burnishing process they used factorial design ($2^3$). $2^3$ factorial designs represent eight-experiments, where the experimental points are located at the vertices of a cube. Four experiments represent an added centre-point to the cube, repeated four times to estimate the pure error. The complete design consists of 12 experiments divided into two blocks, each block containing six experiments and one combined block are considered (trial nos 1 to12)8, 9. This method classifies and identifies the parameters to three different levels (viz. low, center and high) [24]. In this experimentation, twelve tests were carried out at these levels. For each block, the model equations for surface roughness and the surface hardness are obtained by using the analysis of variance technique (ANOVA) and regression coefficient. They develop a mathematical model for obtaining values of surface roughness. J.N. Malleswara Rao et al. (2011) worked on finite element approach for the prediction of Residual stresses in aluminum workpieces Produced by roller burnishing. In this work using numerical approach, compressive residual stress is calculated. Roughness is considered as a triangular asperity in this numerical approach [25]. Before burnishing, the height of the triangle is considered as the roughness of the workpiece. The normal force is acting on the peak of the asperity. Fig 2.1 represents the triangular model for the numerical approach. The depth of deformed layer depends on the yield strength of the material (σy), normal load (Fn), and the asperity angle (α) Commercially available FEA package ANSYS- 12 is used to simulate the analysis process. The burnishing process is modeled as 2 D FEA and the surface roughness is considered as a triangular asperity with included angle of α equal to 800. The height of the triangular asperity is considered as the surface roughness before burnishing.

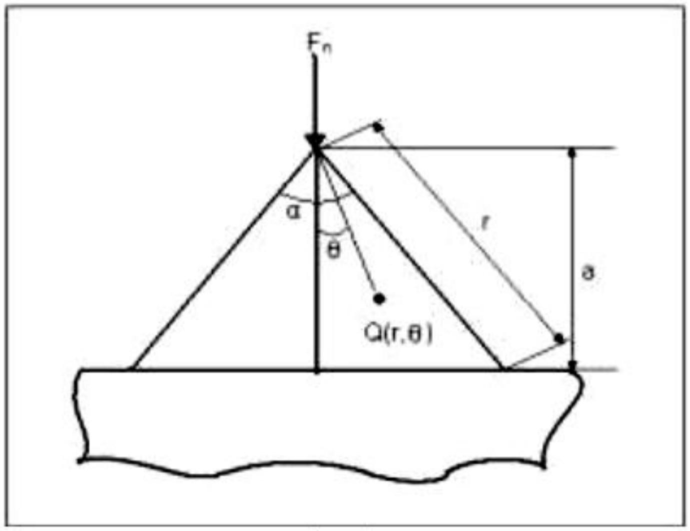


Fig 2.1: Coordinates of a point Q (r, θ) within a triangular asperity [25]

N.S.M. EL-Tayeb et al. (2011) worked on Prediction of Burnishing Surface Integrity using Radial Basis Function. They used artificial neural network (ANN) and radial basic function (RBF) techniques to predict the value of surface roughness [26]. Artificial neural networks are computing elements, which are based on the structure and function of the biological neurons. These networks have nodes or neurons, which are described by difference or differential equations. The nodes are interconnected layer-wise or intra-connected among themselves. Each node in the successive layer receives the inner product of synaptic weights with the outputs of the nodes in the previous layer when the vectors are binary or bipolar, hard-limiting non-linearity is used. When the vectors are analog, a squashed function is used. D. M. Mate et al. (2014),The paper deals with the effect of burnishing process on the Aluminum Alloy material 2014 using Lathe. Surface roughness generated after the turning operation was used to ball burnishing. Improvement in the surface roughness values achieved for tool steel after ball burnishing process was 98.24%. These irregularities causes friction and surface damage which leads to low product life, poor metallurgical properties and overall poor product quality. These processes essentially depend on chip removal to attain the desired surface finish and also, skill and the experience of the operator in handling the process [27].

Srinivas Athreya et al. (2012),Taguchi Method is a statistical approach to optimize the process parameters and improve the quality of components that are manufactured. The objective of this study is to illustrate the procedure adopted in using Taguchi Method to a lathe facing operation. The orthogonal array, signal-to-noise ratio, and the analysis of variance are employed to study the performance characteristics on facing operation. In this analysis, three factors namely speed; feed and depth of cut were considered. Accordingly, a suitable orthogonal array was selected and experiments were conducted. After conducting the experiments the surface roughness was measured and Signal to Noise ratio was calculated. With the help of graphs, optimum parameter values were obtained and the confirmation experiments were carried out. These results were compared with the results of full factorial method [28].

A.B Abdullah et al. (2010), In this study, a sensitivity analysis method was used to identify optimal machining conditions with respect to surface quality. Presently programming Turbo C++ is used to evaluate the property of machined surface with cutting parameter with arbitrary sets of experimental values. Based on the proposed equation and it's differentiate function, the quality of surface roughness can be known clearly through the sensitivities of proper local deviations [29].

The literature review indicates that earlier investigations concentrated on the effect of the ball burnishing process dealing mostly with surface finish and surface hardness with little focus on optimization of the burnishing parameters. The present work aims at methodically studying the effect of process parameters like speed, feed, burnishing force, ball diameter and their interactions on surface roughness by newly developed ball burnishing tool. One factor at a time is carried out to identify the range of parameters used for the experiment. Experiments will be planned according to statistical design of experiments using Taguchi's orthogonal array method, signal to noise ratio method is used to improve the reliability of results. Apart from this, RSM is used to verify the result of optimization which is achieved from Taguchi optimization method.

### 2.5 Dimensional Analysis for mathematical model

A ball burnishing tool consists one or more over-sized balls that are pushed through a hole. Burnishing is a cold working surface finishing process which is carried out on material surfaces to induce compressive residual stresses and enhance surface qualities. The improvements in surface qualities include a reduction in surface roughness, an increase in surface hardness, improvement in grain size, wear resistance, fatigue resistance and corrosion resistance. A burnishing tool typically consists of a hardened sphere which is pressed onto/across the part being processed which results in plastic deformation of asperities into valleys. Mohammadpour et al. (2010) developed a two-dimensional finite element model for orthogonal cutting of AISI 1045 mild steel, and a numerical solution using the FEM. It investigated the effect of cutting speed and feed rate on residual stresses induced after orthogonal cutting [30]. The stress distribution was found to be increasing with respect to cutting speed and feed rate when the experimental and simulation results were compared. N.S.M El-Tayeb et al. (2007) investigated the burnishing process on aluminum 6061 with an interchangeable adapter for both roller and ball burnishing process [8]. The effect of different burnishing parameters like burnishing speed, burnishing force and burnishing tool dimension on the surface qualities and tribological properties were investigated. Partchapol et al. (2007) developed the finite element analysis of ball burnishing process to study the change in properties of work material [31]. The effect of feed rate, flow stress and ball diameter on the surface properties were studied and a detailed explanation was provided. Hamadache et al. (2009) studied the plastic deformation of structural RB40 steel when the ball and roller burnishing were performed. It also investigated the roughness, hardness and wear resistance on RB40 steel [16]. Bouzid et al. (2005) investigated the change in surface roughness of AISI 1042 mild steel after burnishing [5]. A finite element model was formed in which the elastic-plastic behavior of the piece was taken into account to determine the material displacement. The experimental and simulation values were compared and found to be in good correlation. Yung-Chang Yen et al. (2004) studied the change in residual stress values after hard-turning and after roller burnishing process [32]. The corresponding experimental results were compared with the developed FEM models for roller burnishing process from DEFORM 2D and 3D software. The experimental and simulated values were validated.

R. A. Kapgate et al. (2013)The complex phenomenon of wire electrical discharge machining (WEDM) is reducing its utilization to process aluminium silicon carbide with 10% weight metal matrix composite (Al/SiC10% MMC) for industrial applications. This paper presents an experimental investigations and development of mathematical models using dimensional analysis for selection of WEDM process parameters. Sequential classical experimentation technique has been used to perform experiments for triangular, circular and rectangular shape cuts on Al/SiC10% MMC as majority of industrial products are manufactured by these shapes or combinations. An attempt of mini-max principle and linear programming (LPP) has been made to optimize the range bound process parameters for maximizing material removal rate and minimum surface finish to machine Al/SiC10% MMC. The test results proved that MRR and Ra values were significantly influenced by changing important five dimensionless $\pi$ terms. The process parameters grouped in $\pi$ terms were suggested the effective guidelines to the manufacturer for improving productivity by changing any one or all from the available process parameters [33]. N. M. Qureshi et al. (2015) This experimental study focuses on effect of various parameters and optimization of burnishing processes on surface finish of EN8 material during burnishing operation. In industry area use various surface finishing operations such as lapping, honing, etc. which is removal of the material from its surface. In the present experimentation, ball and roller burnishing processes which is plastic deformation are used with varying machining parameters to achieve the desirable surface finish. The experiment is carried out on the CNC machine of a particular job of EN8 material. By use of the Taguchi methodology optimum machining parameters obtained gives improved surface finish [34].

The above literature review shows that there is no mathematical model developed. In present work an effort is made to develop the mathematical model by using the technique of dimensional analysis to correlate speed, feed, force and ball diameter with the surface finish. The model helps in decreasing the number of experiments to be done and it predicts the optimum surface properties. By carefully modeling the ball burnishing process the prediction of the surface characteristic is possible which can be an answer for the time consuming and experimental dependent optimization techniques.

# CHAPTER-3

# METHODOLOGY

## 3.1 Taguchi Method

After the Second World War, the allied forces found that the quality of the Japanese telephone system was extremely poor and totally unsuitable for long-term communication purposes. To improve the system the allied command recommended establishing research facilities in order to develop a state-of-the-art communication system. The Japanese founded the Electrical Communication Laboratories (ECL) with Dr. Genichi Taguchi in charge of improving the R&D productivity and enhancing product quality. He observed that a great deal of time and money was expended on engineering experimentation and testing (Ranjit 1990). Little emphasis was given to the process of creative brainstorming to minimize the expenditure of resources. He noticed that poor quality cannot be improved by the process of inspection, screening, and salvaging. No amount of inspection can put quality back into the product. Therefore, he believed that quality concepts should be based on, and developed around, the philosophy of prevention. Taguchi started to develop new methods to optimize the process of engineering experimentation. He believed that the best way to improve quality was to design and build it into the product. He developed the techniques which are now known as Taguchi Methods. His main contribution lies not in the mathematical formulation of the design of experiments, but rather in the accompanying philosophy. His concepts produced a unique and powerful quality improvement technique that differs from traditional practices. He developed manufacturing systems that were “robust” or insensitive to daily and seasonal variations of environment, machine wear and other external factors. His philosophy had far-reaching consequences, yet it is founded on three very simple concepts. His techniques arise entirely out of these three ideas.

The concepts are:

- Quality should be designed into the product and not inspected into it.
- Quality is better achieved by minimizing the deviation from a target. The product should be so designed that it is immune to uncontrollable environmental factors.
- The cost-quality should be measured as a function of deviation from the standard and the losses should be measured system-wide.

Taguchi viewed quality improvement as an ongoing effort. He continually strived to reduce the variation around the target value. The first step towards improving quality is to achieve the population distribution as close to the target value as possible. To accomplish this, Taguchi designed experiments using specially constructed tables known as " Orthogonal Arrays" (OA). The use of these tables makes the design of experiments very easy and consistent. The Taguchi Method is applied in four steps.

- Brainstorm the quality characteristics and design parameters important to the product/process.
- Design and conduct the experiments.
- Analyse the results to determine the optimum conditions.
- Run a confirmatory test using the optimum conditions.

Taguchi methods start with an assumption that we are designing an engineering system-either a machine to perform some intended function, or a production process to manufacture some product or item. Since we are knowledgeable enough to be designing the system in the first place, we generally will have some understanding of the fundamental processes inherent in that system. Basically, we use this knowledge to make our experiments more efficient. We can skip all the extra effort that might have gone in to investigating interactions that we know do not exist. Without going into the details, it has been shown that this can decrease the level of effort by a factor of ten or twenty and sometimes much more. Another distinction of Taguchi methods is the recognition that there are variables that are under our control and variables that are not under our control.

In Taguchi terms, these are called Control Factors and Noise Factors, respectively. This chapter gives a general introduction to Taguchi Methods. A detailed analysis of results using the method is beyond the scope of the thesis. Hence, we will limit the technique's applicability to the main research topic.

The Taguchi method involves reducing the variation in a process through the robust design of experiments. The overall objective of the method is to produce high-quality product at low cost to the manufacturer. The Taguchi method was developed to maintain that variation. Taguchi developed a method for designing experiments to investigate how different parameters affect the mean and variance of a process performance characteristic that defines how well the process is functioning. The experimental design proposed by Taguchi involves using orthogonal arrays to organize the parameters affecting the process and the levels of which they should be varied. Instead of having to test all possible combinations of the factorial design, the Taguchi method tests pairs of combinations. This allows for the collection of the necessary data to determine which factors most affect product quality with a minimum amount of experimentation, thus saving time and resources. The Taguchi method is best used when there is an intermediate number of a variable (3 to 50), few interactions between variables, and when only a few variables contribute significantly.

The Taguchi arrays can be derived or looked up. Small arrays can be drawn out manually; large arrays can be derived from deterministic algorithms. The arrays are selected by the number of parameters (variables) and the number of levels (states). Analysis of variance on the collected data from the Taguchi design of experiments can be used to select new parameter values to optimize the performance characteristic.

### 3.1.1 An Insight into Orthogonal Arrays (OA) & Taguchi Methods

The technique of laying out the designs of experiments involving numerous factors was first proposed by Sir R. A. Fisher, in the 1920s (Ranjit 1990). The method is popularly known as factorial design of experiments. A full factorial design identifies all possible combinations of a given set of factors. Since most industrial experiments involve a significant number of factors, a full factorial design results may involve a large number of experiments. Factors are the different variable which determines the functionality or performance of a product or system. Factors are:

- Design parameters that influence the performance.
- Input that can be controlled.
- Included in the study for the purpose of determining their influence upon the most desirable performance.

In a heat treatment experiment, for example, a factor can be “cooling rate” or “temperature” etc. Each factor may be set to different levels. Hence for the same experiment the levels can be “slow cooling” and “fast cooling” or “low temperature” and “high temperature” etc. depending on the application.

Taguchi’s approach complements these two important areas. Taguchi constructed a special set of Orthogonal Arrays (OA) to lay out his experiments. By combining existing orthogonal latin squares in a unique manner, Taguchi prepared a new set of standard OAs which could be used for a number of experimental situations. He also devised a standard method for analysis of the results. A single OA may accommodate several experimental situations. Commonly used OAs are available for 2, 3 and 4 levels. The combination of standard experimental design techniques and analysis methods in the Taguchi approach produces consistency and reproducibility.

### 3.1.2 Design of Experiments

The design of experiments (DOE) is a systematic method to determine the relationship between factors affecting a process and the output of that process. In other words, it is used to find cause-and-effect relationships. This information is needed to manage process inputs in order to optimize the output. An understanding of DOE first requires knowledge of some statistical tools and experimentation concepts. Although a DOE can be analyzed in many software programs, it is important for practitioners to understand basic DOE concepts for proper application.

In general usage, the design of experiments (DOE) or experimental design is the design of any information-gathering exercises where variation is present, whether under the full control of the experimenter or not. However, in statistics, these terms are usually used for controlled experiments. Formally planned experimentation is often used in evaluating physical objects, chemical formulations, structures, components, and materials. Other types of study, and their design are discussed in the articles on computer experiments, opinion polls and statistical surveys (which are types of observational study), natural experiments and quasi-experiments (for example, quasi-experimental design). See Experiment for the distinction between these types of experiments or studies.

In the design of experiments, the experimenter is often interested in the effect of some process or intervention (the “treatment”) on some objects (the “experimental units”), which may be people, parts of people, groups of people, plants, animals, etc. The design of experiments is thus a discipline that has very broad application across all the natural and social sciences and engineering.

The general steps involved in the Taguchi Method are as follows:

- Define the process objective, or more specifically, a target value for a performance measure of the process. This may be Surface Roughness (Ra). The target of a process may also be a minimum; for example, the goal may be to minimize the Ra. The deviation in the performance characteristic from the target value is used to define the loss function for the process.

- Determine the design parameters affecting the process. Parameters are variables within the process that affect the performance measure such as speed, feed, force and ball diameter etc. that can be easily controlled. The number of levels that the parameters should be varied at must be specified. For example, speed might be varied to a low and high value of 70 rpm and 410 rpm. Increasing the number of levels to vary a parameter at increases the number of experiments to be conducted.

- Create orthogonal arrays for the parameter design indicating the number of and conditions for each experiment. The selection of orthogonal arrays is based on the number of parameters and the levels of variation for each parameter and will be expounded below.

- Conduct the experiments indicated in the completed array to collect data on the effect on the performance measure.

- Complete data analysis to determine the effect of the different parameters on the performance measure.

See below for a pictorial depiction of these and additional possible steps, depending on the complexity of the analysis.

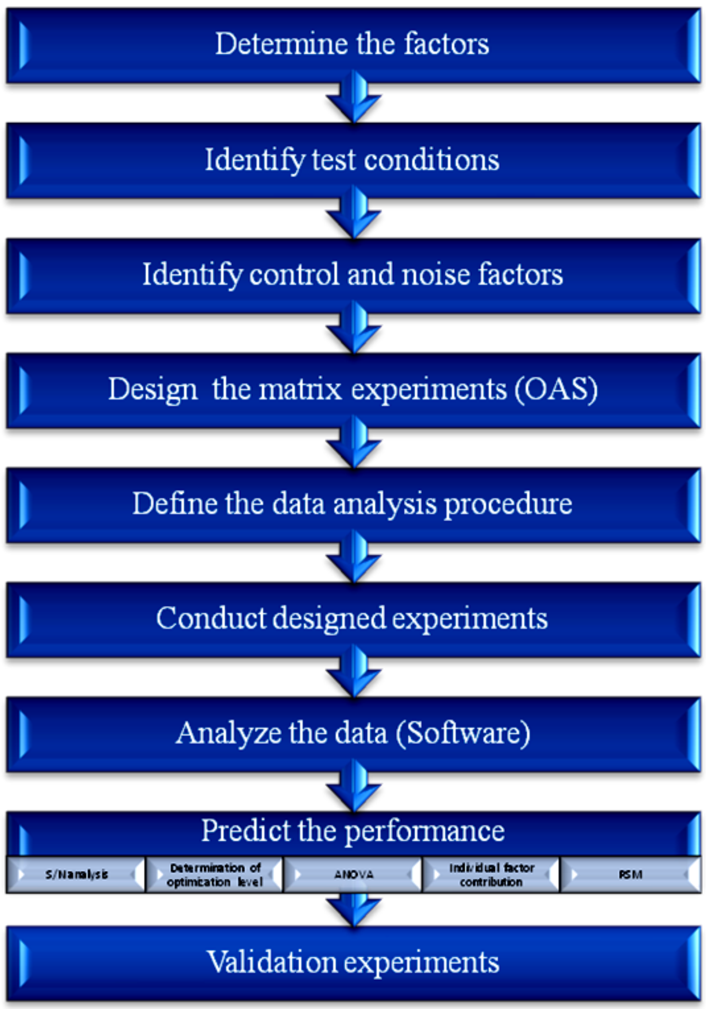


Fig 3.1 General Steps of parameters optimization

The following figure depicted the process (design of experiment, conduct of experiments, selection of burnishing tools etc.) for optimization of burnishing parameter.

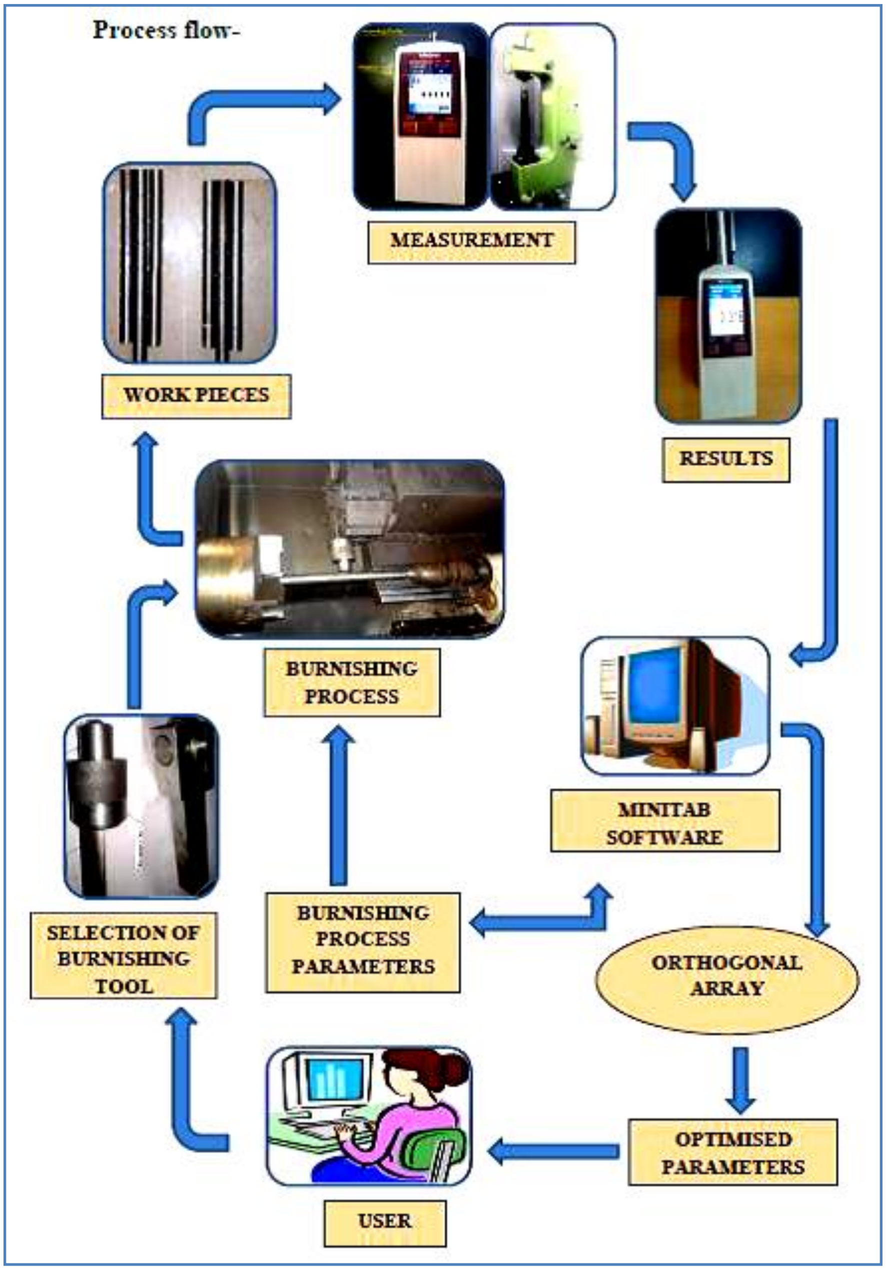


Fig 3.2 Process flow of parameters optimization

### 3.1.3 Signal-to-noise ratio

Signal-to-noise ratio (abbreviated SNR or S/N) is a measure used in science and engineering that compares the level of the desired signal to the level of background noise. It is defined as the ratio of signal power to the noise power, often expressed in decibels. A ratio higher than 1:1 (greater than 0 dB) indicates more signal than noise. While SNR is commonly quoted for electrical signals, it can be applied to any form of signal (such as isotope levels in an ice core or biochemical signaling between cells).

The signal-to-noise ratio, the bandwidth, and the channel capacity of a communication channel are connected by the Shannon–Hartley theorem.

The signal-to-noise ratio is sometimes used informally to refer to the ratio of useful information to false or irrelevant data in a conversation or exchange. For example, in online discussion forums and other online communities, off-topic posts and spam are regarded as “noise” that interferes with the “signal” of appropriate discussion.

### 3.1.4 Signal-to-noise ratio in a Taguchi design

In Taguchi designs, a measure of robustness used to identify control factors that reduce variability in a product or process by minimizing the effects of uncontrollable factors (noise factors). Control factors are those design and process parameters that can be controlled. Noise factors cannot be controlled during production or product use but can be controlled during experimentation. In a Taguchi designed experiment, you manipulate noise factors to force variability to occur and from the results, identify optimal control factor settings that make the process or product robust, or resistant to variation from the noise factors. Higher values of the signal-to-noise ratio (S/N) identify control factor settings that minimize the effects of the noise factors.

Taguchi experiments often use a 2-step optimization process. In step 1 use the signal-to-noise ratio to identify those control factors that reduce variability. In step 2, identify control factors that move the mean to target and have a small or no effect on the signal-to-noise ratio.

The signal-to-noise ratio measures how the response varies relative to the nominal or target value under different noise conditions. You can choose from different signal-to-noise ratios, depending on the goal of your experiment. For static designs, Minitab offers four signal-to-noise ratios:

**Table 3.1 Data characteristic of S/N ratio:**

| Signal-to-noise ratio | Goal of the experiment | Data characteristics | Signal-to-noise ratio formulas |
|---|---|---|---|
| **Larger is better** | Maximize the response | Positive | S/N = -10 *log(Σ(1/$Y^2$)/n) |
| **Nominal is best** | Target the response and you want to base the signal-to-noise ratio on standard deviations only | Positive, zero, or negative | S/N = -10 *log($\sigma^2$) |
| **Nominal is best (default)** | Target the response and you want to base the signal-to-noise ratio on means and standard deviations | Non-negative with an "absolute zero" in which the standard deviation is zero when the mean is zero | $S/N = 10 \times \log((\bar{Y}^2) \div \sigma^2)$<br>The adjusted formula is:<br>$S/N = 10 \times \log((\bar{Y}^2 - s^2 \div n) \div s^2)$ |
| **Smaller is better** | Minimize the response | Non-negative with a target value of zero | S/N = -10 *log(Σ($Y^2$)/n)) |

For Taguchi dynamic designs, Minitab provides one signal-to-noise ratio (and an adjusted formula), which is closely related to the nominal-is-best S/N ratio for static designs.

### 3.1.5 Taguchi Quality Loss Function

The statistical methods developed by Genichi Taguchi to improve the quality of products. Taguchi recognized that in an industrial process it is vital to produce a product on target and that the variation around the mean caused poorly manufactured quality. Taguchi's key argument was that the cost of poor quality goes beyond direct costs to the manufacturer such as reworking or waste costs. Traditionally manufacturers have considered only the costs of quality up to the point of shipping out the product. Taguchi aims to quantify costs over the lifetime of the product. Long-term costs to the manufacturer would include brand reputation and loss of customer satisfaction leading to declining market share. Other costs to the consumer would include costs from low durability, difficulty interfacing with other parts, or the need to build in safety margins. The goal of the Taguchi method is to reduce costs to the manufacturer and to society from variability in manufacturing processes. Taguchi defines the difference between the target value of the performance characteristic of a process, τ, and the measured value, y, as a loss function as shown below.

$$L = k(y - m)^2 \tag{1}$$

Where:

L = Loss (currency)

y = Quality Characteristic (diameter, concentration, etc)

m = Target Value for y

k = Constant (defined below)

$$k = \frac{Ao}{\Delta_0^2}$$

Where:

$A_0$ = Consumer Loss (currency)

$\Delta_0$ = Maximum Deviation from Target Allowed by Consumer

There are three characteristics used to define the quality loss function:

1. Nominal–the-Best Characteristic
2. Smaller-the-Better Characteristic
3. Larger-the-Better Characteristic

Each of these characteristic types is defined by a different set of equations, which is different from the general form of the loss function equation.

### 3.1.6 Nominal–the-Best Characteristic

For a nominal characteristic, there is a defined target value for the product which has to be achieved. There is a specified upper and lower limit, with the target specification being the middle point. Quality is, in this case, is defined in terms of deviation from the target value.

The equation (1) used to describe the loss function of one unit of product:

$$L = k(y - m)^2$$

$$L = k(MSD)$$

(y –m) = Mean Squared Deviation

As the output value (y) deviates from the target value (m) increasing the mean squared deviation, the loss (L) increases. There is no loss when the output value is equal to the target value (y = m).

Again, we can develop a performance measure or S/N ratio which approximation the expected quality loss

$$\eta_{\text{nominal is better}} = -10\log\left(\frac{1}{n-1}\sum_{i=1}^{n}(Y_i-\bar{Y})^2\right) \quad (2)$$

**3.1.7 Lower-the- Better**

In the case of Smaller-the-Better characteristic, the ideal target value is defined as zero. An example of this characteristic is the minimization of heat losses in a heat exchanger. Minimizing this characteristic as much as possible would produce a more desirable product.

The equation used to describe the loss function of one unit of product:

$$L = ky^2 \quad (4)$$

Where:

k = Proportionality Constant

y = Output Value

The proportionality constant (k) for the Smaller-the-Better characteristic can be determined as follows:

$$k = \frac{A_0}{y_0^2}$$

Where:

$A_0$ = Consumer Loss (currency)

$y_0$ = Maximum Consumer Tolerated Output Value

The loss is minimized as the output value is minimized.

Again, we can develop a performance measure or S/N ratio which minimize the expected quality loss

$$\eta_{\text{lower is better}} = -10\log\left(\frac{1}{n}\sum_{i=1}^{n} Y_i^2\right) \tag{5}$$

### 3.1.8 Higher-the-Better

The Larger–the-Better characteristic is just the opposite of the Smaller-the-Better characteristic. For this characteristic type, it is preferred to maximize the result, and the ideal target value is infinity. An example of this characteristic is maximizing the product yield from a process.

The equation used to describe the loss function of one unit of product:

$$L = \frac{k}{y_0^2} \tag{6}$$

Where:

k = Proportionality Constant

$y_0$ = Minimum Consumer Tolerated Output Value

The proportionality constant (k) for the Larger-the-Better characteristic can be calculated by using the equation given for the Smaller-the-Better proportionality constant. The only difference between the two is the definition of $y_0$.

This characteristic is the opposite of the Lower-the-Better characteristic, as the loss is minimized as the output value is maximized.

Again, we can develop a performance measure or S/N ratio which minimize the expected quality loss

$$\eta_{\text{higher is better}} = -10\log\left(\frac{1}{n}\sum_{i=1}^{n} \frac{1}{Y_i^2}\right) \tag{7}$$

### 3.2 Response surface (RSM)

Response surface Methodology (RSM) is a collection of mathematical and statistical techniques for empirical model building. By careful design of experiments, the objective is to optimize a response (output variable) which is influenced by several independent variables (input variables). An experiment is a series of tests, called runs, in which changes are made in the input variables in order to identify the reasons for changes in the output response. Originally, RSM was developed to model experimental responses (Box and Draper, 1987), and then migrated into the modeling of numerical experiments. The difference is the type of error generated by the response. In physical experiments, inaccuracy can be due, for example, to measurement errors while, in computer experiments, numerical noise is a result of incomplete convergence of iterative processes, round-off errors or the discrete representation of continuous physical phenomena (Giunta et al., 1996; van Campen et al., 1990, Toropov et al., 1996). In RSM, the errors are assumed to be random. The application of RSM to design optimization is aimed at reducing the cost of expensive analysis methods (e.g. finite element method or CFD analysis) and their associated numerical noise. With smooth functions that improve the convergence of the optimization process because they reduce the effects of noise and they allow for the use of derivative-based algorithms. Venter et al. (1996) have discussed the advantages of using RSM for design optimization applications.

Two important models are commonly used in RSM. These are special cases of model (1) and include the first-degree model (d =1),

$$y = \beta_0 + \sum_{i=1}^{k} \beta_i x_i + \epsilon \tag{8}$$

and the second-degree model (d =2)

$$y = \beta_0 + \sum_{i=1}^{k} \beta_i x_i + \sum_{i<j}\sum \beta_{ij} x_i x_j + \sum_{i=1}^{k} \beta_{ii} x_i^2 + \epsilon. \tag{9}$$

The purpose of considering a model such as (1) is threefold:

- To establish a relationship, albeit approximate, between y and $x_1,x_2,...,x_k$ that can be used to predict response values for given settings of the control variables.
- To determine, through hypothesis testing, the significance of the factors whose levels are represented by $x_1,x_2,...,x_k$.
- To determine the optimum settings of $x_1,x_2,...,x_k$ that result in the maximum (or minimum) response over a certain region of interest.

### 3.2.1Objectives and Typical Applications of RSM

Response surface is useful in the solution of many types of industrial problems. Generally, these problems fall into three categories:

- Mapping a Response Surface over a Particular Region of Interest. Consider the chemical process. Normally, this process would operate at a particular setting of reaction time and reaction temperature. However, some changes to these normal operating levels might occasionally be necessary, perhaps to produce a product that meets other specific customer requirement. If the true unknown response function has been approximated over a region around the current operating conditions with a suitably fitted response surface (say a second-order surface), then the process engineer can predict in advance the changes in yield that will result from any readjustments to time and temperature.
- Optimization of the Response. In the industrial world, a very important problem is determining the conditions that optimize the process. A second-order model could then be used to approximate the yield response in a narrow region around point and from an examination of this approximating response surface, the optimum levels or condition for time and temperature could be chosen.

- Selection of Operating Conditions to Achieve Specifications or Customer requirements. In most response surface problems there are several responses that must in some sense be simultaneously considered.

### 3.2.2 RSM and the Philosophy of Quality Improvement

During the last few decades, industrial organizations in the United States and Europe have become keenly interested in quality and process improvement. Statistical methods, including statistical process control (SPC) and design of experiments, play a key role in this activity. Quality improvement is most effective when it occurs early in the product and process development cycle. It is very difficult and often expensive to manufacture a poorly designed product. Industries such as semiconductors and electronics, aerospace, automotive, biotechnology and pharmaceuticals, medical devices, chemical, and process industries are all examples where experimental design methodology has resulted in shorter design and development time for new products, as well as products that are easier to manufacture, have higher reliability, have enhanced field performance, and meet or exceed customer requirements. RSM is an important branch of experimental design in this regard. RSM is a critical technology in developing new processes, optimizing their performance, and improving the design and/or formulation of new products. It is often an important concurrent engineering tool, in that product design, process development, quality, manufacturing engineering, and operations personnel often work together in a team environment to apply RSM. The objectives of quality improvement, including reduction of variability and improved product and process performance, can often be accomplished directly using RSM.

### 3.3 Dimensional Analysis

In engineering and science, dimensional analysis is the analysis of the relationships between different physical quantities by identifying their fundamental dimensions (such as length, mass, time, and electric charge) and units of measure (such as miles vs. kilometers, or pounds vs. kilograms vs. grams) and tracking these dimensions as calculations or comparisons are performed. Converting from one-dimensional unit to another is often somewhat complex. Dimensional analysis, or more specifically the factor-label method, also known as the unit-factor method, is a widely used technique for such conversions using the rules of algebra.

The concept of physical dimension was introduced by Joseph Fourier in 1822.Physical quantities that are commensurable have the same dimension; if they have different dimensions, they are incommensurable. For example, it is meaningless to ask whether a kilogram is less, the same, or more than an hour.

Any physically meaningful equation (and likewise any inequality and in the equation) will have the same dimensions on the left and right sides, a property known as "dimensional homogeneity". Checking this is a common application of dimensional analysis. Dimensional analysis is also routinely used as a check on the plausibility of derived equations and computations. It is generally used to categorize types of physical quantities and units based on their relationship to or dependence on other units.

The dimensional analysis offers a method for reducing complex physical problems to the simplest (that is, most economical) form prior to obtaining a quantitative answer. Bridgman (1969) explains it thus: "The principal use of dimensional analysis is to deduce from a study of the dimensions of the variables in any physical system certain limitations on the form of any possible relationship between those variables. The method is of great generality and mathematical simplicity". At the heart of the dimensional analysis is the concept of similarity. In physical terms, similarity refers to some equivalence between two things or phenomena that are actually different. For example, under some very particular conditions, there is a direct relationship between the forces acting on a full-size aircraft and those on a small-scale model of it. The question is, what are those conditions, and what is the relationship between the forces? Mathematically, similarity refers to a transformation of variables that leads to a reduction in the number of independent variables that specify the problem. Here the question is what kind of transformation works? The dimensional analysis addresses both these questions. Its main utility derives from its ability to contract or make more succinct, the functional form of physical relationships. A problem that at first looks formidable may sometimes be solved with little effort after dimensional analysis.

In problems so well understood that one can write down in mathematical form all the governing laws and boundary conditions, and only the solution is lacking, similarity can also be inferred by normalizing all the equations and boundary conditions in terms of quantities that specify the problem and identifying the dimensionless groups that appear in the resulting dimensionless equations. This is an inspectional form of similarity analysis. Since inspectional analysis can take advantage of the problem's full mathematical specification, it may reveal a higher degree of similarity than a "blind" (less informed) dimensional analysis and in that sense prove more powerful. Dimensional analysis is, however, the only option in problems where the equations and boundary conditions are not completely articulated and always useful because it is simple to apply and quick to give insight. Some of the basic ideas of similarity and dimensional analysis had already surfaced in Fourier's work in the nineteenth century's first quarter, but the subject received more methodical attention only toward the close of that century, notably in the works of Lord Rayleigh, Reynolds, Maxwell, and Froude in England, and Carvallo, Vaschy and a number of other scientists and engineers in France (Macagno1971). By the 1920's the principles were essentially in place: Buckingham's now ubiquitous $\pi-$ theorem had appeared (Buckingham, 1914), and Bridgman had published the monograph which still remains the classic in the field (Bridgman, 1922, 1931). Since then, the literature has grown prodigiously. Applications now include aerodynamics, hydraulics, ship design, propulsion, heat and mass transfer, combustion, mechanics of elastic and plastic structures, fluid-structure interactions, electromagnetic theory, radiation, astrophysics, underwater and underground explosions, nuclear blasts, impact dynamics, and chemical reactions and processing (see for example Sedov, 1959, Baker et al, 1973, Kurth, 1972, Lokarnik, 1991), and also biology (McMahon & Bonner, 1983) and even economics (de Jong, 1967). Most applications of dimensional analysis are not in question, no doubt because they are well supported by experimental facts. The debate over the method's theoretical-philosophical underpinnings, on the other hand, has never quite stopped festering (e.g. Palacios, 1964). Mathematicians tend to find in the basic arguments a lack of rigor and are tempted to redefine the subject in their own terms (e.g. Brand, 1957), while physicists and engineers often find themselves uncertain about the physical meanings of the words in terms of which the analysis cast. The problem is that dimensional analysis is based on ideas that originate at such a substantial point in science that most scientists and engineers have lost touch with them. To understand its principles, we must return to some of the very fundamental concepts in science. Dimensional analysis is rooted in the nature of the artifices we construct in order to describe the physical world and explain its functioning in quantitative terms. Einstein (1933) has said, "Pure logical thinking cannot yield us any knowledge of the empirical world; all knowledge starts from experience and ends in it. Propositions arrived at by purely logical means are completely empty as regards reality".

This treatise is an attempt to explain dimensional analysis by tracing it back to its physical foundations. We will clarify the terms used in the dimensional analysis, explain why and how it works, remark on its utility, and discuss some of the difficulties and questions that typically arise in its application. One single (unremarkable) application in mechanics will be used to illustrate the procedure and its pitfalls. The procedure is the same in all applications, a great variety of which may be found in the references and in the scientific literature at large.

### 3.3.1The steps of dimensional analysis

The premise of dimensional analysis is that the form of any physically significant equation must be such that the relationship between the actual physical quantities remains valid independent the magnitudes of the base units. Dimensional analysis derives the logical consequences of this premise. Suppose we are interested in some particular physical quantity $Q_0$ that is a “dependent variable” in a well defined physical process or event. By this, we mean that once all the quantities that define the particular process or event are specified, the value of $Q_0$ follows uniquely.

**Step 1: The independent variables**

The first and most important step in dimensional analysis is to identify a complete set of independent quantities

$Q_1 ... Q_n$ that determine the value of $Q_0$,

$Q_0 = f(Q_1, Q_2, ... , Q_n)$

A set $Q_1 ... Q_n$ is complete if, once the values of the members are specified, no other quantity can affect the value of $Q_0$, and independent if the value of each member can be adjusted arbitrarily without affecting the value of any other member. Starting with a correct set $Q_1 ... Q_n$ is as important in the dimensional analysis as it is in mathematical physics to start with the correct fundamental equations and boundary conditions. If the starting point is wrong, so is the answer. We defer to the question of how correct set is to be identified.

The relationship expressed symbolically in above equation is the result of the physical laws that govern the phenomenon of interest. It is our premise that its form must be such that, once the values $Q_1 ... Q_n$ are specified, the equality holds regardless of the sizes of the base units in terms of which the quantities are measured. The steps that follow derive the consequences of this premise.

**Step 2: Dimensional considerations**

Next, we list the dimensions of the dependent variable $Q_0$ and the independent variables $Q_1...Q_n$. As we have discussed, the dimension of a quantity depends on the type of system of units and we must specify at least the type the system of units before we do this. For example, if we use a system and are dealing with a purely mechanical problem, all quantities have dimensions of the form

$[Q_i]=L^{li}M^{mi}t^{Ti}$

where the exponents

$l_i$, $m_i$ and $T_i$ are dimensionless numbers that follow from each quantity's definition. We now pick from the complete set of physically independent variables

$Q_1...Q_n$ a complete, dimensionally independent subset $Q_1...Q_k(k\leq n)$, and express the dimension of each of the remaining independent variables $Q_{k+1}...Q_n$ and the dependent variable Q0 as a product of powers of $Q_1...Q_k$.

All physical quantities have dimensions which can be expressed as products of powers of the set of base dimensions. Alternatively, it is possible to express the dimension of one quantity as a product of powers of the dimensions of other quantities which are not necessarily base quantities. A subset $Q_1...Q_k$ of the set $Q_1...Q_n$ is dimensionally independent if none of its members has a dimension that can be expressed in terms of the dimensions of the remaining members. And complete if the dimensions of all the remaining quantities

$Q_{k+1}...Q_n$ of the full set can be expressed in terms of the dimensions of the subset $Q_1...Q_k$. Since equation mentioned in step1 is dimensionally homogeneous, the dimension of the dependent variable Q0 is also expressible in terms of the dimensions of $Q_1...Q_k$. The dimensionally independent subset $Q_1...Q_k$ is picked by trial and error. Its members may be picked in different ways, but the number k of dimensionally independent quantities in the full set $Q_1...Q_n$ is unique to the set, and cannot exceed the number of base dimensions which appear in the dimensions the quantities in that set. For example, if the dimensions of $Q_1...Q_n$ involve only length, mass, and time, then $k\leq 3$. Having chosen a complete, dimensionally independent subset $Q_1...Q_k$, we express the dimensions of $Q_0$ and the remaining quantities $Q_{k+1}...Q_n$ in terms of the dimensions of $Q_1...Q_k$. These will have the form

$[Q_i]=[Q_1^{Ni1}Q_2^{Ni2}...Q_k^{Nik}]$

if $i>k$ or $i=0$. The exponents $N_{ij}$ are dimensionless real numbers and can in most cases be found quickly by inspection, although a formal algebraic method can be used. The formal procedure can be illustrated with an example where length, mass and time are the only base quantities, in which case all dimensions have the form of above equation. Let us take $Q_1$, $Q_2$,and $Q_3$ as the complete dimensionally independent subset. Equating the dimension given by equation which is mentioned at *step 2* with that of above equation, we obtain three equations

$l_i=\sum N_{ij}l_j$

$J=1\sim3$

$m_i=\sum N_{ij}m_j$

$J=1\sim3$

$T_i=\sum N_{ij}T_j$

$J=1\sim3$

Which can be solved for the three unknowns Ni1, Ni2,and Ni3.

**Step 3: Dimensionless variables**

We now define dimensionless forms of the n-k remaining independent variables by dividing each one by the product of powers of $Q_1...Q_k$

which has the same dimension,

$\Pi i=Q_{k+i}/Q_1^{N(k+i)1}Q_2^{N(k+i)2}...Q_k^{N(k+i)k}$

where i=1, 2,..., n-k, and a dimensionless form of the dependent variable Q0,

$\Pi_0 = Q_0 / Q_1^{N01} Q_2^{N02} ... Q_k^{N0k}$

**Step 4: The end game and Buckingham's –theorem**

An alternative form of equation which mentioned at *step1* is

$\Pi_0 = f(Q_1, Q_2, ...., Q_k; \Pi_1, \Pi_2, ..., \Pi_{n-k})$

in which all quantities are dimensionless except $Q_1...Q_k$. The values of the dimensionless quantities are independent of the sizes of the base units. The values of $Q_1...Q_k$, on the other hand, do depend on base unit size. They cannot be put into dimensionless form since they are (by definition) dimensionally independent of each other. From the principle that any physically meaningful equation must be dimensionally homogeneous, that is, valid independent of the sizes of the base units, it follows that $Q_1...Q_k$ must, in fact, be absent from equation above that is,

$\Pi_0 = f(\Pi_1, \Pi_2, ..., \Pi_{n-k})$

This equation is the final result of the dimensional analysis.

### 3.3.2 Methods of Dimensional Analysis

There are two methods of dimensional analysis.

1. Rayleigh's method
2. Buckingham's (Π– theorem) method

**Rayleigh's method**

Rayleigh's method of analysis is adopted when a number of parameters or variables are less (3 or 4 or 5).

**Methodology**

$X_1$ is a function of $X_2, X_3, X_4 .......... X_n$

Then it can be written as $X_1 = f(X_2, X_3, X_4 ... X_n)$

In this equation, $X_1$ is a dependent variable, while $X_2$, $X_3$, $X_4$………. $X_n$ are independent variables.

The above-noted equation may be expressed as

$X_1 = K (X_2^a, X_3^b, X_4^c \ldots)$

Taking dimensions for all the quantities $[X_1] = [X_2]^a [X_3]^b [X_4]^c \ldots\ldots$

Dimensions for quantities on left-hand side as well as on the right-hand side are written and using the concept of Dimensional Homogeneity a, b, c …. can be determined. The dimensionless parameters are then formed by grouping together the variables with like powers.

Then, $X_1 = K X_2^{a.} X_3^b. X_4^c$

In which K is a dimensionless constant which may be determined either from physical characteristics of the problem or from experimental measurements.

**Buckingham's Π Method**

This method of analysis is used when numbers of variables are more.

**Buckingham's Π Theorem**

If there are n – variables in a physical phenomenon and those n-variables contain 'm' dimensions, then the variables can be arranged into (n-m) dimensionless groups called Π terms.

**Explanation**

If $f (X_1, X_2, X_3 \ldots X_n) = C$ and variables can be expressed using m dimensions then.

$f(\Pi_1, \Pi_2, \Pi_3, \ldots\ldots\ldots, \Pi_{n-m}) = C_1$

Where, $\Pi_1$, $\Pi_2$, $\Pi_3$ ……..... are dimensionless groups.

Thus a total number of variable is n and this entire variable may be completely described by m fundamental dimensions of either M-L-T or F-L-T systems (i.e. m=3). Therefore there are (n-m) is dimensionless $\Pi$ terms

Each $\Pi$ term being dimensionless, the dimensional homogeneity can be used to get each $\Pi$ term.

$\Pi_1 = X_1^{a1} X_2^{b1} X_3^{c1} \ldots X_m^{m1} X_{m+1}$

$\Pi_2 = X_1^{a2} X_2^{b2} X_3^{c2} \ldots X_m^{m2} X_{m+2}$

…………………………………………………………

$\Pi_{n-m} = X_1^{a(n-m)} X_2^{b(n-m)} X_3^{c(n-m)} \ldots X_m^{m(n-m)} X_n$

The final general equation for the phenomenon may then be obtained by expressing any one of $\Pi$ terms as a function of the others as

$\Pi_1 = f_1 (\Pi_{2,} \Pi_3 \ldots \Pi_{n-m})$

$\Pi_2 = f_2 (\Pi_{1,} \Pi_3 \ldots \Pi_{n-m})$

$\Pi_3 = f_3 (\Pi_{1,} \Pi_2 \ldots \Pi_{n-m})$

Or any other desired relationship may be obtained.

### 3.3.3 Selecting Repeating Variables

The objectives of selecting repeating variables are to:

1. Avoid taking the quantity required as the repeating variable.
2. Repeating variables put together should not form a dimensionless group.
3. No two repeating variables should have same dimensions.
4. Repeating variables can be selected from each of the following properties.
   a. Geometric property: Length, height, width, area
   b. Flow property: Velocity, Acceleration, and Discharge
   c. Fluid property: Mass density, Viscosity, Surface tension

### 3.4 Models in Research

A model in OR is a simplified representation of an operation, or is a process in which only the basic aspects or the most important features of a typical problem under investigation are considered. The objective of a model is to identify significant factors and interrelationships. The reliability of the solution obtained from a model depends on the validity of the model representing the real system. A good model must possess the following characteristics:

(i)It should be capable of taking into account, the new formulation without having any changes in its frame.

(ii) Assumptions made in the model should be as small as possible.

(iii) Variables used in the model must be less in number ensuring that it is simple and coherent.

(iv) It should be open to the parametric type of treatment.

(v) It should not take much time in its construction for any problem.

#### 3.4.1 Advantages of a Model

There are certain significant advantages gained when using a model these are:

(i)Problems under consideration become controllable through a model.

(ii) It provides a logical and systematic approach to the problem.

(iii) It provides the limitations and scope of an activity.

(iv) It helps in finding useful tools that eliminate duplication of methods applied to solve problems.

(v) It helps in finding solutions for research and improvements in a system.

(vi) It provides an economic description and explanation of either the operation or the systems they represent.

### 3.4.2 Models of Structure

Mathematical models are most abstract in nature. They employ a set of mathematical symbols to represent the components of the real system. These variables are related together by means of mathematical equations to describe the behavior of the system. The solution of the problem is then obtained by applying well-developed mathematical techniques to the model. We can also define a mathematical model as consisting of:

- Decision variables, which are the unknowns to be determined by the solution to the model.
- Constraints to represent the physical limitations of the system
- An objective function
- An optimal solution to the model is the identification of a set of variable values which are feasible (satisfy all the constraints) and which lead to the optimal value of the objective function.

An optimization model seeks to find values of the decision variables that optimize (maximize or minimize) an objective function among the set of all values for the decision variables that satisfy the given constraints.

### 3.4.3 Terminology

Solution:

The set of values of decision variables (j = 1, 2, ... n) which satisfy the constraints is said to constitute a solution to meet the problem.

Feasible Solution:

The set of values of decision variables $X_j$ (j = 1, 2, ... n) which satisfy all the constraints and non-negativity conditions of a linear programming problem simultaneously is said to constitute the feasible solution to that problem.

Infeasible Solution:

The set of values of decision variables $X_j$ (j = 1, 2, ... n) which do not satisfy all the constraints and non-negativity conditions of the problem is said to constitute the infeasible solution to that linear programming problem.

Basic Solution:

For a set of m simultaneous equations in n variables (n > m), a solution obtained by setting (n –m) variables equal to zero and solving for remaining m variables is called a basic feasible solution.

The variables which are set to zero are known as non-basic variables and the remaining m variables which appear in this solution are known as basic variables:

Basic Feasible Solution:

A feasible solution to LP problem which is also the basic solution is called the basic feasible solution. Basic feasible solutions are of two types;

(a) Degenerate: A basic feasible solution is called degenerate if the value of at least one basic variable is zero.

(b) Non- degenerate: A basic feasible solution is called non-degenerate‘ if all values of m basic variables are non-zero and positive.

Optimum Basic Feasible Solution:

A basic feasible solution which optimizes (maximizes or minimizes) the objective function value of the given LP problem is called an optimum basic feasible solution.

Unbounded Solution:

A basic feasible solution which optimizes the objective function of the LP problem indefinitely is called unbound solution

### 3.5 Introduction to Linear Programming

Linear programming deals with the optimization (maximization or minimization) of a function of variables known as objective functions. It is subject to a set of linear equalities and /or inequalities known as constraints. Linear programming is a mathematical technique, which involves the allocation of limited resources in an optimal manner, on the basis of a given criterion of optimality. In this section properties of Linear Programming Problems (LPP) are discussed. The graphical method of solving an LPP is applicable where two variables are involved. The most widely used method for solving LP problems consisting of any number of variables is called Simplex method.

#### 3.5.1 Formulation of LP Problems

The procedure for mathematical formulation of an LPP consists of the following steps:

**Step 1** To write down the decision variables of the problem.

**Step 2** To formulate the objective function to be optimized (Maximized or Minimized) as a linear function of the decision variables.

**Step 3** To formulate the other conditions of the problem such as resource limitation, market constraints, interrelations between variables etc., as linear in equations or equations in terms of the decision variables.

**Step 4** To add the non-negativity constraint from the considerations so that the negative values of the decision variables do not have any valid physical interpretation.

The objective function, the set of constraint and the non-negative constraint together form a Linear programming problem.

### 3.5.2 General Formulation of LPP

The general formulation of the LPP can be stated as follows:

In order to find the values of n decision variables $x_1$ , $x_2$ , $x_3$ ,....., $x_n$ to maximize or minimize the objective function.

$Z = c_1x_1 + c_2 x_2 + ..... + c_n x_n$

and also satisfy m-constraints

$a_{11} x_1+a_{12}x_2+....a_{1n}x_n \leq=\geq b_1$

$a_{21} x_1+a_{22}x_2+....a_{2n}x_n \leq=\geq b_2$

.......

$a_{m1} x_1+a_{m2}x_2+....a_{mn}x_n \leq=\geq b_m$

Where constraints may be in the form of inequality < or > or even in the form an equation (=) and finally satisfy the nonnegative restrictions

$x_1\geq$, $x_2\geq$, $x_3\geq$,.....$x_n\geq 0$.

### 3.6 Sum of squares

The sum of squares represents a measure of variation or deviation from the mean. It is calculated as a summation of the squares of the differences from the mean. The calculation of the total sum of squares considers both the sum of squares of the factors and from randomness or error.

### 3.6.1 Sum of squares in ANOVA

In an analysis of variance (ANOVA), the total sum of squares helps express the total variation that can be attributed to various factors. For example, you do an experiment to test the effectiveness of three laundry detergents.

The total sum of squares = treatment sum of squares (SST) + sum of squares of the residual error (SSE)

The treatment sum of squares is the variation attributed to, or in this case between, the laundry detergents. The sum of squares of the residual error is the variation attributed to the error.

Converting the sum of squares into mean squares by dividing by the degrees of freedom lets you compare these ratios and determine whether there is a significant difference due to detergent. The larger this ratio is, the more the treatments affect the outcome.

### 3.6.2 Sum of squares in regression

In regression, the total sum of squares helps express the total variation of the y's. For example, you collect data to determine a model explaining overall sales as a function of your advertising budget.

The total sum of squares = regression sum of squares (SSR) + sum of squares of the residual error (SSE)

$$\Sigma(y - \overline{y})^2 = \Sigma(\hat{y} - \overline{y})^2 + \Sigma(y - \hat{y})^2 \tag{10}$$

The regression sum of squares is the variation attributed to the relationship between the x's and y's, or in this case between the advertising budget and your sales. The sum of squares of the residual error is the variation attributed to the error. By comparing the regression sum of squares to the total sum of squares, you determine the proportion of the total variation that is explained by the regression model (R2, the coefficient of determination). The larger this value is the better the relationship explaining sales as a function of advertising budget.

### 3.7 Analysis of variance (ANOVA)

Analysis of variance (ANOVA) is similar to regression in that it is used to investigate and model the relationship between a response variable and one or more independent variables. However, analysis of variance differs from the regression in two ways: the independent variables are qualitative (categorical), and no assumption is made about the nature of the relationship (that is, the model does not include coefficients for variables). In effect, analysis of variance extends the two-sample t-test for testing the equality of two population means to a more general null hypothesis of comparing the equality of more than two means, versus them not all being equal. Several of MINITAB's ANOVA procedures, however, allow models with both qualitative and quantitative variables.

Like so many of our inference procedures, ANOVA has some underlying assumptions which should be in place in order to make the results of calculations completely trustworthy. They include:

(i)Subjects are chosen via a simple random sample.

(ii) Within each group/population, the response variable is normally distributed.

(iii) While the population means may be different from one group to the next, the population standard deviation is the same for all groups.

Analysis of Variance (ANOVA) is a computational technique to quantitatively estimate the relative contribution, which each controlled parameter makes to the overall measured response and expressing it as a percentage. ANOVA uses the S/N ratio responses for these calculations. The basic idea of ANOVA is that the total sum of squares of the standard deviation is equal to the sum of squares of the standard deviation caused by each parameter.

#### 3.7.1 One-way and two-way ANOVA models

- **One-way analysis of variance** tests the equality of population means when classification is by one variable. The classification variable, or factor, usually has three or more levels (one-way ANOVA with two levels is equivalent to a t-test), where the level represents the treatment applied. For example, if you conduct an experiment where you measure the durability of a product made by one of three methods, these methods constitute the levels. The one-way procedure also allows you to examine differences among means using multiple comparisons.

- **Two-way analysis of variance** performs an analysis of variance for testing the equality of population means when the classification of treatments is by two variables or factors. In two-way ANOVA, the data must be balanced (all cells must have the same number of observations) and factors must be fixed. If you wish to specify certain factors to be random, use Balanced ANOVA if your data are balanced; use General Linear Models if your data are unbalanced or if you wish to compare means using multiple comparisons.

### 3.7.2 One-Way ANOVA Table

When you have an experiment with a one-way layout, you compute the F statistic using a one-way ANOVA table. Below is how the ANOVA table is calculated.

**Table 3.2 Table of One-Way ANOVA**

| Source of Variation | d.f. | SS | MS | $F_0$ |
|---|---|---|---|---|
| Factor A (between groups) | a-1 | $SSA = \sum_{i=1}^{a} n_i \left(\bar{y}_{i.} - \bar{y}_{..}\right)^2$ | $MSA = \frac{SSA}{(a-1)}$ | $\frac{MSA}{MSE}$ |
| Error (within groups) | N-a | $SSE = SST - SSA$ | $MSE = \frac{SSE}{(N-a)}$ | |
| Total | N-1 | $SST = \sum_{i=1}^{a} \sum_{j=1}^{n} \left(y_{ij} - \bar{y}_{..}\right)^2$ | | |

Some helpful definitions for this table:

a = the number of levels for a factor

i = level of factor

j = trial at a given level

$n_i$ = the number of trials at the $i^{th}$ factor level

$y_{ij}$ = the response value at ith factor level and the $j^{th}$ trial

$y_{..}$ = overall mean of data

$y_{i.}$ = the mean at $i^{th}$ factor level

The $F_{crit}$ for such an analysis is $F_{alpha, a-1, N-a}$ and can be found in the F table.

### 3.7.3 Two-Way ANOVA Table

When you have an experiment with a two-way layout, use a two-way ANOVA table to calculate the F statistic. Below is how the two-way ANOVA table is calculated.

**Table 3.3 Table of Two-Way ANOVA**

| Source of Variation | d.f. | SS | MS | $F_0$ |
|---|---|---|---|---|
| Factor A (between groups) | a-1 | $SSA = \sum_{i=1}^{a} n_i \left(\bar{y}_{i.} - \bar{y}_{..}\right)^2$ | $MSA = \frac{SSA}{(a-1)}$ | $\frac{MSA}{MSE}$ |
| Factor B (between groups) | b-1 | $SSB = \sum_{j=1}^{b} n_j \left(\bar{y}_{.j} - \bar{y}_{..}\right)^2$ | $MSB = \frac{SSB}{(b-1)}$ | $\frac{MSB}{MSE}$ |
| Error (within groups) | (a-1)(b-1) | $SSE = SST - SSA - SSB$ | $MSE = \frac{SSE}{(a-1)(b-1)}$ | |
| Total | N-1 | $SST = \sum_{i=1}^{a}\sum_{j=1}^{n} \left(y_{ij} - \bar{y}_{..}\right)^2$ | | |

Some helpful definitions for this table:

a = the number of levels for the first factor

b = the number of levels for the second factor

i = level of the first factor

j = level of the second factor

$n_i$ = the number of trials at the $i^{th}$ factor level for the first factor

$n_j$ = the number of trials at the $j^{th}$ factor level for the second factor

$y_{ij}$ = the response value at ith and $j^{th}$ factor levels

$y_{..}$ = overall mean of data

$y_{i.}$ = the mean at the $i^{th}$ factor level for the first factor

$y_{.j}$ = the mean at the $j^{th}$ factor level for the second factor

The $F_{crit}$ for such an analysis is $F_{alpha,\ a-1,\ (a-1)(b-1)}$ for factor A and $F_{alpha,\ b-1,\ (a-1)(b-1)}$ for factor B. This can be found in the F table.

The percentage contribution, P of the process parameters on the roughness can be calculated as:

$$P = \frac{SS_d}{SS_T} \tag{11}$$

# CHAPTER-4

## EXPERIMENTAL SETUP

### 4.1 Design of Ball Burnishing Tool

A magnetic holding ball burnishing tool is designed to carry out the experimental work on Mild Steel using high chromium high carbon ball. The ball is holder is supported elastically by a pre-calibrated spring, which could apply the required force when pressed onto the workpiece surface. The use of the spring is important for reducing sticking due to friction between the ball and the workpiece. The amount of spring compression with relation to the applied vertical force ($P_Y$).

The burnishing tool consists of parts namely ball holder, circular casing, magnet, spring, and bearing. The design is made in consideration with the parameters in the work. The experimental work is planned to conduct mainly considering four different parameters and burnishing force is one among the parameters. So, the force is measured by means of spring deflections in the tool.

To design burnishing tool, it is essential to calculate the normal burnishing force for given condition.

P.N Patel et al (2014) shown the Normal Burnishing Force equation to evaluate the spring stiffness [17]

Normal Burnishing Force: $P_Y = \pi \varepsilon H R^2$ (12)

Where

$\varepsilon$ = h/R= Depth of penetration/Ball Radius = Relative depth of penetration = 0.002-0.003

R = Ball Radius = 1.5, 2.5, 3.5, 4.5, 5.5 mm

H = Vickers Hardness of Work Material

For mild steel, H = Vickers Hardness = 140

(a) R = Ball Radius = 1.5

Normal Burnishing Force $P_Y = \pi\varepsilon HR^2$

$P_Y = (3.14)\ x(0.0025)x(140)x(1.5)^2$

$P_Y = 2.47Kgf = 24\ N$

Feed force: $P_X = (0.04\ to\ 0.20)\ x\ P_Y = 0.1x24 = 2.4\ N$

b) R = Ball Radius = 2.5

Normal Burnishing Force $P_Y = \pi\varepsilon HR^2$

$P_Y = (3.14)x(0.0025)x(140)x(2.5)^2$

$P_Y = 6.87\ Kgf = 67\ N$

Feed force: $P_X = (0.04\ to\ 0.20)\ x\ P_Y = 0.1x69 = 6.90\ N$

c) R = Ball Radius = 3.5

Normal Burnishing Force $P_Y = \pi\varepsilon HR^2$

$P_Y = (3.14)x(0.0025)x(140)x(3.5)^2$

$P_Y = 13.46\ Kgf = 132\ N$

Feed force: $P_X = (0.04\ to\ 0.20)\ x\ P_Y = 0.1x132 = 13.20\ N$

(d) R = Ball Radius = 4.5

Normal Burnishing Force $P_Y = \pi\varepsilon HR^2$

$P_Y = (3.14)x(0.0025)x(140)x(4.5)^2$

$P_Y = 22.25\ Kgf = 218\ N$

Feed force: $P_X = (0.04\ to\ 0.20)\ x\ P_Y = 0.1x218 = 21.8\ N$

(e) R = Ball Radius = 5.5

Normal Burnishing Force $P_Y = \pi\varepsilon HR^2$

$P_Y = (3.14)x(0.0025)x(140)x(5.5)^2$

$P_Y = 33.24\ Kgf = 326\ N$

Feed force: $P_X = (0.04\ to\ 0.20)\ x\ P_Y = 0.1x326 = 32.60\ N$

Taking maximum normal burnishing force: $P_{Y(max)} = 1.2\ x\ 326 = 391.2\ N$

### 4.1.1 Spring Stiffness

The spring used in these work is of EN 9 material, which helps to measures burnishing force by means of spring deflections in the square casing.

Length of Spring $L_F$ = 44 mm

Spring Diameter D = 18 mm

Spring Wire Diameter d = 2.5 mm

Number of Turns N = n + 2 = 9+2 = 11

Solid Length Ls = N x d = 27.5 mm

Deflection of spring is provided by rotating the Force

$L_F$ = Nxd + $\delta_{max}$+ (N-1) x 0.5 mm (13)

$\delta_{max}$ = 11.5 mm

In this expression, the clearance between two adjacent coils is taken as 0.5 mm (when compressed)

Maximum Deflection of spring $\delta_{max}$ = 11.5 mm

Maximum Normal Burnishing Force $P_Y$ = 391.2 N

Stiffness $K_1 = P_Y /\delta = 391.2/11.5x10^{-3}$ = 34,017 N/M

Selected spring deflection is 0.25mm for 1kgf force

So, available Stiffness of spring we selected for the study K = $9.81/0.25x10^{-3}$ = 39,240 N/M

As ($K_1$< K), The Design of spring is safe.

### 4.1.2 Adjustable Chuck with Magnet

An adjustable chuck of 80 mm dia and 70 mm length is used for this burnishing tool. The chuck has three jaws separated by $120^0$ angle. In each of these jaws, a slot is prepared according to a small size bearing (Bearing No: 504 Z, Dimension: OD= 12 mm & Thickness = 4 mm). Inside the chuck, a cylindrical magnet is placed which hold the ball of various diameter on the bearing. This burnishing tool is designed in such a way that it produces less friction between the ball and the workpiece.

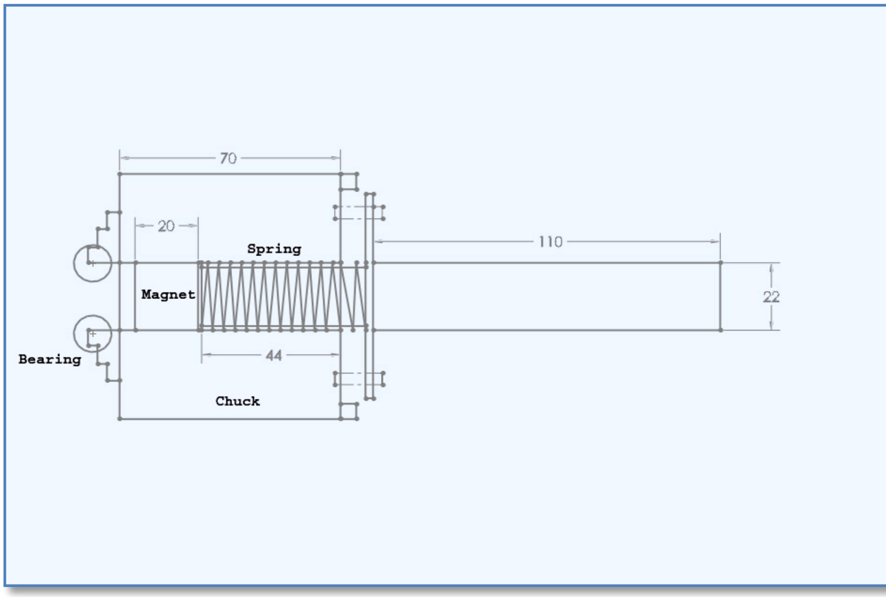


Fig. 4.1 CAD Drawing of Ball Burnishing Tool

Fig. 4.2 Ball Burnishing Tool

## 4.2 Workpiece

Commercially available Mild steel is used in the present experimental work. Work piece diameter is 45mm. Burnishing experiments are conducted on turned Mild workpiece. First, the workpiece is held in 3- jaw chuck of a lathe and facing operation is completed on both sides and centre drilling is completed on both the faces. Then, the workpiece is held in between centers of lathe and it is driven by the lathe dog. A high-speed steel (H. S.S.) single point cutting tool is fixed in the tool post of the lathe and workpiece is turned to have 75mm length and 45 mm diameter of 25 pcs workpiece. In actual experiments, the different parameters were applied on the workpiece.

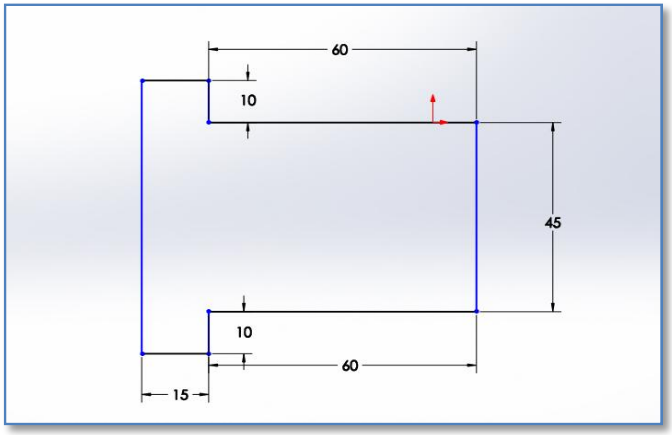


Fig. 4.3 CAD Drawing of Workpiece

Fig. 4.4 Workpiece for Experiment

### 4.3 Setup of Burnishing tool and workpiece

The Ball burnishing tool is mounted on the Lathe. The experimental set up with ball burnishing tool is shown in Fig. 4.1. It consists of the parts: (1) Three jaw chuck, (2) Live center, (3) Dead center, (4) Mild steel workpiece, (5) Ball Burnishing tool, (6) Hand wheel for cross slide of lathe

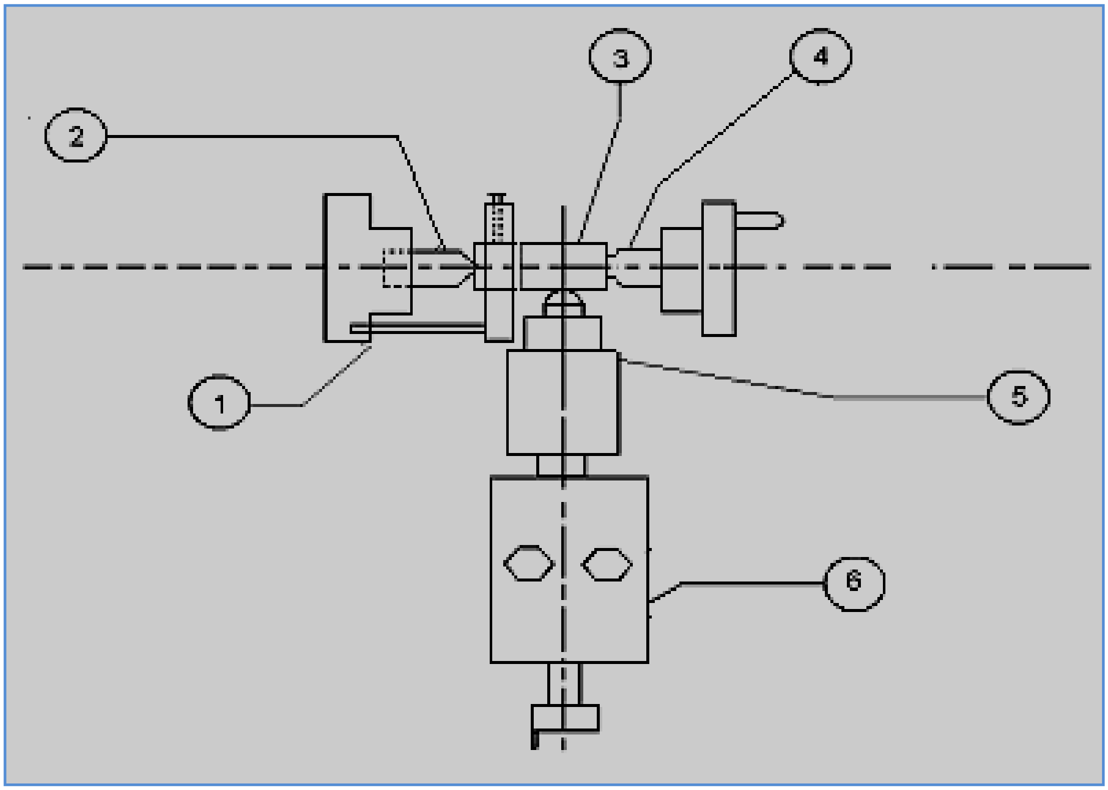


Fig. 4.5 Experimental setup with Ball burnishing tool

## 4.4 Setup of Roughness measuring device

The values of mean surface roughness (Ra) before and after burnishing were measured by using Roughness tester ‘‘Surftest SJ-210’’, Mitutoyo. For each workpiece, the average Ra was obtained by three measurements conducted along the longitudinal direction at different positions.

Specification of Roughness tester:

Measuring range: 17.5 mm

Measuring speed: 0.25, 0.5, 0.75 mm/s

Detector range: 360 µm (-200 µm to + 160 µm )

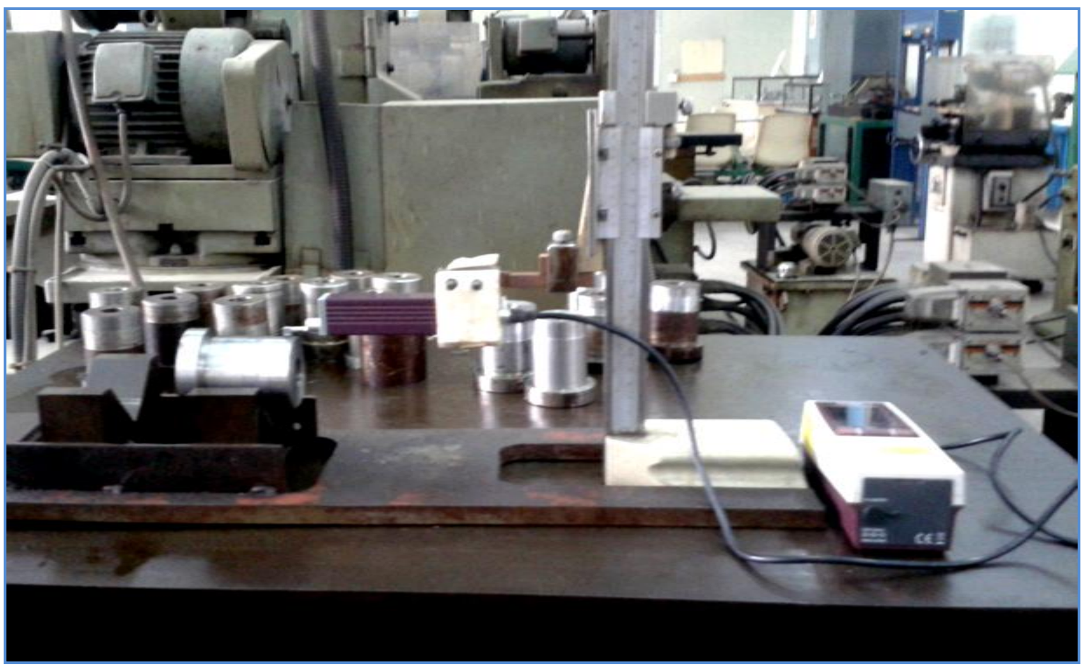

Fig. 4.6 Experimental setup of Surface Roughness measuring device

### 4.5 Array Selector

Orthogonal arrays are a special standard experimental design that requires only a small number of experimental trials to find the main factors effects on output. Before selecting an orthogonal array, the minimum number of experiments to be conducted is to be fixed based on the formula below

$$N\ Taguchi = 1 + NV\ (L - 1) \tag{14}$$

N Taguchi = Number of experiments to be conducted

$NV_{Max}$ =Maximum Number of parameters

L = Number of levels

From formula

$NV_{Max}$ = 6 and L = 5, Hence

N Taguchi = 1+ 6 (5-1) = 25

Hence at least 25 experiments are to be conducted. Based on this orthogonal array (OA) is to be selected which has at least 25 rows i.e., 25 experimental runs

L25 is applicable for Number of parameter 2 ~ 6.

In this work, Number of parameter 4 and level 5. So we will use L25 Table

| Number of Levels | Number of Parameters (P) | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 | 19 | 20 | 21 | 22 | 23 | 24 | 25 | 26 | 27 | 28 | 29 | 30 | 31 |
| 2 | L4 | L4 | L8 | L8 | L8 | L8 | L12 | L12 | L12 | L12 | L16 | L16 | L16 | L16 | L32 | L32 | L32 | L32 | L32 | L32 | L32 | L32 | L32 | L32 | L32 | L32 | L32 | L32 | L32 | L32 |
| 3 | L9 | L9 | L9 | L18 | L18 | L18 | L18 | L27 | L27 | L27 | L27 | L27 | L36 | L36 | L36 | L36 | L36 | L36 | L36 | L36 | L36 | L36 | | | | | | | | |
| 4 | L'16 | L'16 | L'16 | L'16 | L'32 | L'32 | L'32 | L'32 | L'32 | | | | | | | | | | | | | | | | | | | | | |
| 5 | L25 | L25 | L25 | L25 | L25 | L50 | L50 | L50 | L50 | L50 | L50 | | | | | | | | | | | | | | | | | | | |

### 4.6 Design of Experiment (DOE)

In this study, four burnishing parameters were selected for optimizing the burnishing process using Taguchi method. The examined burnishing parameters include: (1) Burnishing speed, (2) Force and (3) Feed rate (4) Ball diameter. Other parameters such as a number of burnishing passes and penetration depth are considered constant in the course of this study. For each parameter, 5(five) levels were considered. According to Taguchi method with 4 (four) independent parameters, 25 experiments will be conducted. Five coded levels will be used for each parameter and MINITAB software has used for data analysis. The materials used in this work are made of Mild Steel with dimension L=75mm and D=45mm. A burnishing tool device designed and made to perform the burnishing experiments. The device is in the form of a small lathe chuck of 80mm diameter with adjustable 3(three) jaws separated by 120° angle. In each of these jaws, a 504Z No. bearing of OD =12mm T = 4mm can be mounted. The tool is designed in such a way that it can be simply mounted onto a Center Lathe Machine. A Conventional Lathe machine will be used for this study.

**Table 4.1 Parameter of burnishing process**

| **Factors** | **Symbols** | **Levels** | | | | |
|---|---|---|---|---|---|---|
| | | **-2** | **-1** | **0** | **1** | **2** |
| Speed (n), rpm | X1 | 70 | 155 | 240 | 325 | 410 |
| Force ($P_Y$), N | X2 | 24 | 78 | 132 | 186 | 240 |
| Feed (f), mm/rev | X3 | 0.1 | 0.18 | 0.26 | 0.34 | 0.42 |
| Ball diameter (d), mm | X4 | 3 | 5 | 7 | 9 | 11 |
| Ball material ($H_B$), N/mm$^2$(Brinell scale) | - | 653 | - | - | - | - |
| Relative Penetration (ε) | - | 0.0025 | | | | |
| No. of Passes | - | 1 | - | - | - | - |
| Burnishing Condition | - | Dry | - | - | - | - |

$Z-2\sigma <= X <= Z+2\sigma$

Z = Mean

σ = Standard Deviation

[-2, -1, 0, 1, 2] using the following transformation equations:

$X1 = (n-240)/85$

$X2 = (P_Y-132)/54$

$X3 = (f-0.26)/0.08$

X4 = (d-7)/2

Where X1, X2, X3and X4 denotes burnishing speed, force, feed rate and ball diameter, respectively.

**Table 4.2 Taguchi L25 ($5^4$) Table**

| Runs | Factor A | Factor B | Factor C | Factor D |
|---|---|---|---|---|
| **1** | 1 | 1 | 1 | 1 |
| **2** | 1 | 2 | 2 | 2 |
| **3** | 1 | 3 | 3 | 3 |
| **4** | 1 | 4 | 4 | 4 |
| **5** | 1 | 5 | 5 | 5 |
| **6** | 2 | 1 | 2 | 3 |
| **7** | 2 | 2 | 3 | 4 |
| **8** | 2 | 3 | 4 | 5 |
| **9** | 2 | 4 | 5 | 1 |
| **10** | 2 | 5 | 1 | 2 |
| **11** | 3 | 1 | 3 | 5 |
| **12** | 3 | 2 | 4 | 1 |
| **13** | 3 | 3 | 5 | 2 |
| **14** | 3 | 4 | 1 | 3 |
| **15** | 3 | 5 | 2 | 4 |
| **16** | 4 | 1 | 4 | 2 |
| **17** | 4 | 2 | 5 | 3 |
| **18** | 4 | 3 | 1 | 4 |
| **19** | 4 | 4 | 2 | 5 |
| **20** | 4 | 5 | 3 | 1 |
| **21** | 5 | 1 | 5 | 4 |
| **22** | 5 | 2 | 1 | 5 |
| **23** | 5 | 3 | 2 | 1 |
| **24** | 5 | 4 | 3 | 2 |
| **25** | 5 | 5 | 4 | 3 |

# CHAPTER-5

## RESULTS AND DISCUSSION

### 5.1 Result of surface quality

In this research design the experiments using Taguchi L25 table for four burnishing factors, and optimization the level of these factors analyze by Signal-to-noise ratio. However, Taguchi method cannot judge the contribution of individual factor, so ANOVA is used to determine the individual contribution of each factor. Apart from this, to test the optimum level of parameter that is achieved by Taguchi method RSM is applied. A second order mathematical model is used to analyze the RSM. The coefficient of this model is evaluated by regression analysis by MINITAB16 software. For the purpose of verifying the optimum level of Taguchi method, two optimum factors have been remained constant and roughness is changed with another two parameters in the RSM model. Then analyze the graphical representation of this model. As per objectives of this research to establish the relationship among the factors an empirical data based model is formulated by dimensional analysis of the physical burnishing parameters.

**Table 5.1 Four independent Burnishing Parameters**

| Factors | Levels | | | | |
|---|---|---|---|---|---|
| | -2 | -1 | 0 | 1 | 2 |
| Speed (n), rpm | 70 | 155 | 240 | 325 | 410 |
| Force ($P_Y$), N | 24 | 78 | 132 | 186 | 240 |
| Feed (f), mm/rev | 0.1 | 0.18 | 0.26 | 0.34 | 0.42 |
| Ball diameter (d), mm | 3 | 5 | 7 | 9 | 11 |

According to DOE, 4 (four) independent parameters, 25 experiments had been conducted with the workpiece, which had 2.85 μm initial roughness and following results were obtained.

**Table 5.2 Experimental design matrix and result of surface quality**

| Exp No. | Speed, rpm | | Force, N | | Feed, mm/rev | | Ball Diameter | | Responses |
|---|---|---|---|---|---|---|---|---|---|
| | Coded Value | Actual Value | Coded Value | Actual Value | Coded Value | Actual Value | Coded Value | Actual Value | Ra, μm |
| **1** | -2 | 70 | -2 | 24 | -2 | 0.1 | -2 | 3 | 1.60 |
| **2** | -2 | 70 | -1 | 78 | -1 | 0.18 | -1 | 5 | 1.13 |
| **3** | -2 | 70 | 0 | 132 | 0 | 0.26 | 0 | 7 | 2.68 |
| **4** | -2 | 70 | 1 | 186 | 1 | 0.34 | 1 | 9 | 2.26 |
| **5** | -2 | 70 | 2 | 240 | 2 | 0.42 | 2 | 11 | 1.73 |
| **6** | -1 | 115 | -2 | 24 | -1 | 0.18 | 0 | 7 | 2.23 |
| **7** | -1 | 115 | -1 | 78 | 0 | 0.26 | 1 | 9 | 1.14 |
| **8** | -1 | 115 | 0 | 132 | 1 | 0.34 | 2 | 11 | 1.22 |
| **9** | -1 | 115 | 1 | 186 | 2 | 0.42 | -2 | 3 | 2.12 |
| **10** | -1 | 115 | 2 | 240 | -2 | 0.1 | -1 | 5 | 1.55 |
| **11** | 0 | 240 | -2 | 24 | 0 | 0.26 | 2 | 11 | 1.23 |
| **12** | 0 | 240 | -1 | 78 | 1 | 0.34 | -2 | 3 | 1.47 |
| **13** | 0 | 240 | 0 | 132 | 2 | 0.42 | -1 | 5 | 2.95 |
| **14** | 0 | 240 | 1 | 186 | -2 | 0.1 | 0 | 7 | 1.20 |
| **15** | 0 | 240 | 2 | 240 | -1 | 0.18 | 1 | 9 | 1.72 |
| **16** | 1 | 325 | -2 | 24 | 1 | 0.34 | -1 | 5 | 2.55 |
| **17** | 1 | 325 | -1 | 78 | 2 | 0.42 | 0 | 7 | 1.26 |
| **18** | 1 | 325 | 0 | 132 | -2 | 0.1 | 1 | 9 | 1.50 |
| **19** | 1 | 325 | 1 | 186 | -1 | 0.18 | 2 | 11 | 1.73 |
| **20** | 1 | 325 | 2 | 240 | 0 | 0.26 | -2 | 3 | 2.15 |
| **21** | 2 | 410 | -2 | 24 | 2 | 0.42 | 1 | 9 | 2.72 |
| **22** | 2 | 410 | -1 | 78 | -2 | 0.1 | 2 | 11 | 1.16 |
| **23** | 2 | 410 | 0 | 132 | -1 | 0.18 | -2 | 3 | 2.50 |
| **24** | 2 | 410 | 1 | 186 | 0 | 0.26 | -1 | 5 | 2.30 |
| **25** | 2 | 410 | 2 | 240 | 1 | 0.34 | 0 | 7 | 2.20 |

### 5.2 Parameters optimization by Taguchi Method

The Full Factorial Design requires a large number of experiments to be carried out. It becomes laborious and complex if the number of factors increases. To overcome this problem Taguchi suggested a specially designed method called the use of the orthogonal array to study the entire parameter space with a lesser number of experiments to be conducted. Taguchi thus, recommends the use of the loss function to measure the performance characteristics that are deviating from the desired target value. The value of this loss function is further transformed into signal-to-noise (S/N) ratio. Usually, there are three categories of the performance characteristics to analyze the S/N ratio. They are nominal-the-best, larger-the-better, and smaller-the-better. For the higher performance of product, a low surface roughness is always desired. Hence, the response parameter surface roughness (Ra) and less effect on specimen diameter (d) after burnishing means accuracy that has been categorized as 'lower is better' type problem and the signal to noise ratio, in this case, has been calculated as equation (5).

$$\eta_{\text{lower is better}} = -10\log\left(\frac{1}{n}\sum_{i=1}^{n} Y_i^2\right)$$

Also, for better performance a first-rate microhardness (HRB) are required. Hence, this response parameter has been categorized as 'higher is better' and the signal to noise ratio, in this case, has been calculated as equation (7).

$$\eta_{\text{higher is better}} = -10\log\left(\frac{1}{n}\sum_{i=1}^{n} \frac{1}{Y_i^2}\right)$$

Where, n= Sample Size, and Y= Responses in that run.

The following table shows the S/N ratio using 'lower is better' formula for resulted surface roughness of each experiment to determine the optimum level of burnishing parameters.

**Table 5.3 S/N ratio table with respect to response**

| Exp No. | Responses | |
|---|---|---|
| | Ra, µm | S/N Ratio |
| **1** | 1.60 | -4.08 |
| **2** | 1.13 | -1.06 |
| **3** | 2.68 | -8.56 |
| **4** | 2.26 | -7.08 |
| **5** | 1.73 | -4.76 |
| **6** | 2.23 | -6.97 |
| **7** | 1.14 | -1.14 |
| **8** | 1.22 | -1.73 |
| **9** | 2.12 | -6.53 |
| **10** | 1.55 | -3.80 |
| **11** | 1.23 | -1.80 |
| **12** | 1.47 | -3.34 |
| **13** | 2.95 | -9.40 |
| **14** | 1.20 | -1.58 |
| **15** | 1.72 | -4.71 |
| **16** | 2.55 | -8.13 |
| **17** | 1.26 | -2.00 |
| **18** | 1.50 | -3.52 |
| **19** | 1.73 | -4.76 |
| **20** | 2.15 | -6.65 |
| **21** | 2.72 | -8.69 |
| **22** | 1.16 | -1.29 |
| **23** | 2.50 | -7.96 |
| **24** | 2.30 | -7.23 |
| **25** | 2.20 | -6.85 |

By average the S/N ratio of each factor from the above table for every level (+2 to -2) we get the following table

**Table 5.4 Average S/N Ratios for each factor**

| Level | Speed, rpm | Force, N | Feed, mm/rev | Ball Diameter, mm |
|---|---|---|---|---|
| **-2** | -5.11 | -5.93 | -2.85* | -5.71 |
| **-1** | -4.03* | -1.76* | -5.09 | -5.92 |
| **0** | -4.17 | -6.23 | -5.07 | -5.19 |
| **1** | -5.01 | -5.44 | -5.43 | -5.02 |
| **2** | -6.40 | -5.35 | -6.28 | -2.87* |
| **Rank** | -1 | -1 | -2 | 2 |

The factor levels corresponding to the highest S/N ratio were chosen to optimize the condition.

**Best set of variables:** Speed (n) = 155 rpm, Force ($P_Y$) = 78 N, Feed (f) = 0.1, Ball Diameter (d) = 11 mm

By using these parameters experiment had conducted and 0.8 µm roughness was achieved.

### 5.3 Effect of burnishing parameters on roughness according to ANOVA

Taguchi Method cannot judge and determine effect of individual parameters on entire process while percentage contribution of individual parameters can be well determined using ANOVA

**Table 5.5 Average Responses (Ra) for each factor**

| Level | Speed, rpm | Force, N | Feed, mm/rev | Ball Diameter, mm |
|---|---|---|---|---|
| **-2** | 1.88 | 2.06 | 1.40 | 1.97 |
| **-1** | 1.65 | 1.23 | 1.86 | 2.09 |
| **0** | 1.71 | 2.17 | 1.90 | 1.91 |
| **1** | 1.84 | 1.92 | 1.94 | 1.87 |
| **2** | 2.18 | 1.87 | 2.16 | 1.41 |
| **Over all Mean** | 1.852 | 1.85 | 1.852 | 1.85 |

**Sum of Square** of Factors (Speed, Force, Feed, and Ball Diameter) will be evaluated by “Between group” and **Sum of Square** of Error will be evaluated by "Within group”

$$SS_{Total} = \sum [(Ra) - (\overline{\overline{Ra}})]^2 \tag{15}$$

$= (1.6-1.852)^2 + (1.13-1.852)^2 + (2.68-1.852)^2 + (2.26-1.852)^2 + (1.73-1.852)^2 + (2.23-1.852)^2 + (1.14-1.852)^2 + (1.22-1.852)^2 + (2.12-1.852)^2 + (1.55-1.852)^2 + (1.23-1.852)^2 + (1.47-1.852)^2 + (2.95-1.852)^2 + (1.2-1.852)^2 + (1.72-1.852)^2 + (2.55-1.852)^2 + (1.26-1.852)^2 + (1.5-1.852)^2 + (1.73-1.852)^2 + (2.15-1.852)^2 + (2.72-1.852)^2 + (1.16-1.852)^2 + (2.5-1.852)^2 + (2.3-1.852)^2 + (2.2-1.852)^2$

$SS_{Total} = 7.884$

$$SS_{Factor} = n \sum [(\overline{Ra})_{(i)} - (\overline{\overline{Ra}})_{(i)}]^2 \tag{16}$$

Here, n= Level No. = 5, i = -2,-1, 0, 1, 2

$SS_{speed} = 5[(1.88-1.852)^2 + (1.65-1.852)^2 + (1.71-1.852)^2 + (1.84-1.852)^2 + (2.18 -1.852)^2]$

$= 0.846$

$SS_{Force} = 5[(2.06-1.85)^2 + (1.23-1.85)^2 + (2.17-1.85)^2 + (1.92-1.85)^2 + (1.87-1.85)^2]$

$= 2.681$

$SS_{Feed} = 5[(1.40-1.852)^2 + (1.86-1.852)^2 + (1.90-1.852)^2 + (1.94-1.852)^2 + (2.16 -1.852)^2]$

$= 1.546$

$SS_{Ball\ Dia} = 5[(1.97-1.85)^2 + (2.09-1.85)^2 + (1.91-1.85)^2 + (1.87-1.85)^2 + (1.41 - 1.85)^2]$

$= 1.348$

$$SS_{Error} = SS_{Total} - SS_{speed} - SS_{force} - SS_{feed} - SS_{ball\ dia} \quad (17)$$

$$= 7.884 - (0.846+2.681+1.546+1.348)$$

$$= 1.463$$

**Table 5.6 ANOVA for Reponses (Ra)**

| Source of Variance | Sum of Square | DF | Variance (Mean Square) | F ratio (Variance ratio) | P (%) |
|---|---|---|---|---|---|
| Speed, rpm | 0.846 | 4 | 0.211 | 1.159 | 10.73% |
| Force, N | 2.681 | 4 | 0.670 | 3.681 | 34.00% |
| Feed, mm/rev | 1.546 | 4 | 0.386 | 2.120 | 19.61% |
| Ball Diameter, mm | 1.348 | 4 | 0.337 | 1.851 | 17.09% |
| Error | 1.463 | 8 | 0.182 | 1 | 18.55% |
| Total | 7.884 | 24 | | | 100% |

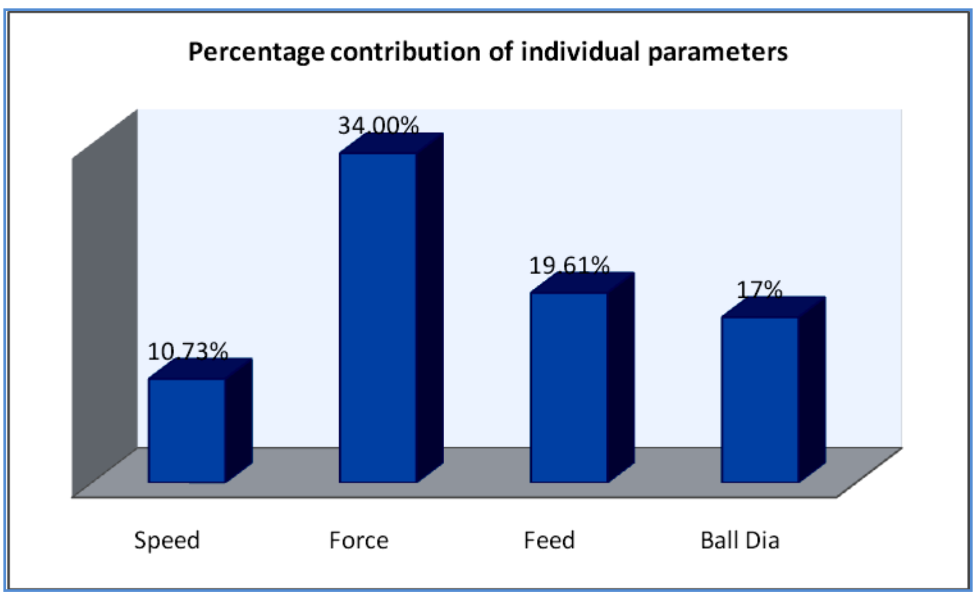


Fig. 5.1 Percentage of contribution of individual parameter

### 5.4 Surface Response Method

From the Table 4.1 X1, X2, X3and X4 denotes burnishing speed, force, feed rate and ball diameter, respectively. Table 5.2 shows the arrangements and the results of the 25 experiments that were performed based on Taguchi design. Using these results in order to find the relationship between the surface roughness of Mild Steel and the four ball burnishing parameters X1, X2, X3and X4, a second-order mathematical model was used to the form as equation (9):

$$Y = b_0 + b_1X1 + b_2X2 + b_3X3 + b_4X4 + b_{11}X1^2 + b_{22}X2^2 + b_{33}X3^2 + b_{44}X4^2 + b_{12}X1X2 + b_{13}X1X3 + b_{23}X2X3 + b_{34}X3X4$$

Where the b terms are the regression coefficients and y is Ra . The estimated coefficients b by MINITAB Software as shown in **Appendix-1**

**Table 5.7 Analysis of Variance of RSM**

| Source | DF | SS | MS | F | P |
|---|---|---|---|---|---|
| Regression | 11 | 3.8447 | 0.3495 | 1.12 | 0.415 |
| Residual Error | 13 | 4.0395 | 0.3107 | | |
| Total | 24 | 7.8842 | | | |

| Source | DF | Seq SS |
|---|---|---|
| X1 | 1 | 0.3026 |
| X2 | 1 | 0.0444 |
| X3 | 1 | 1.2577 |
| X4 | 1 | 0.8924 |
| X1xX1 | 1 | 0.5092 |
| X2xX2 | 1 | 0.0510 |
| X3xX3 | 1 | 0.0844 |
| X4xX4 | 1 | 0.3774 |
| X1xX2 | 1 | 0.2128 |
| X1xX3 | 1 | 0.1003 |
| X3xX4 | 1 | 0.0124 |
| Total | 11 | 3.8447 |

The regression equation is

$$Ra = 1.86 + 0.0695\,X1 + 0.0273\,X2 + 0.136\,X3 - 0.179\,X4 + 0.0936\,X1^2 + 0.0567\,X2^2 - 0.0573\,X3^2 - 0.0992\,X4^2 - 0.0452\,X1X2 - 0.045\,X1X3 - 0.0166\,X3X4 \quad (18)$$

### 5.4.1 Validation of RSM Model

The purpose of the validation is to evaluate the accuracy of the prediction model with the experimental data**.** In this work, the prediction errors are defined as follows

$$\text{Prediction error} = \frac{\text{Pred. result} - \text{Exp. result}}{\text{Exp. result}} \times 100\% \quad (19)$$

**Table 5.8 Ra (Experimented) and Ra (RSM model)**

| Exp No. | Speed, rpm | | Force, N | | Feed, mm/rev | | Ball Diameter | | Responses | |
|---|---|---|---|---|---|---|---|---|---|---|
| | Coded Value | Actual Value | Coded Value | Actual Value | Coded Value | Actual Value | Coded Value | Actual Value | Ra, µm | Ra (RSM Model) |
| **1** | -2 | 70 | -2 | 24 | -2 | 0.1 | -2 | 3 | 1.6 | 1.30 |
| **2** | -2 | 70 | -1 | 78 | -1 | 0.18 | -1 | 5 | 1.13 | 1.81 |
| **3** | -2 | 70 | 0 | 132 | 0 | 0.26 | 0 | 7 | 2.68 | 2.10 |
| **4** | -2 | 70 | 1 | 186 | 1 | 0.34 | 1 | 9 | 2.26 | 2.14 |
| **5** | -2 | 70 | 2 | 240 | 2 | 0.42 | 2 | 11 | 1.73 | 1.96 |
| **6** | -1 | 115 | -2 | 24 | -1 | 0.18 | 0 | 7 | 2.23 | 1.73 |
| **7** | -1 | 115 | -1 | 78 | 0 | 0.26 | 1 | 9 | 1.14 | 1.59 |
| **8** | -1 | 115 | 0 | 132 | 1 | 0.34 | 2 | 11 | 1.22 | 1.22 |
| **9** | -1 | 115 | 1 | 186 | 2 | 0.42 | -2 | 3 | 2.12 | 2.17 |
| **10** | -1 | 115 | 2 | 240 | -2 | 0.1 | -1 | 5 | 1.55 | 1.71 |
| **11** | 0 | 240 | -2 | 24 | 0 | 0.26 | 2 | 11 | 1.23 | 1.28 |
| **12** | 0 | 240 | -1 | 78 | 1 | 0.34 | -2 | 3 | 1.47 | 1.96 |
| **13** | 0 | 240 | 0 | 132 | 2 | 0.42 | -1 | 5 | 2.95 | 2.01 |
| **14** | 0 | 240 | 1 | 186 | -2 | 0.1 | 0 | 7 | 1.2 | 1.44 |
| **15** | 0 | 240 | 2 | 240 | -1 | 0.18 | 1 | 9 | 1.72 | 1.69 |
| **16** | 1 | 325 | -2 | 24 | 1 | 0.34 | -1 | 5 | 2.55 | 2.41 |
| **17** | 1 | 325 | -1 | 78 | 2 | 0.42 | 0 | 7 | 1.26 | 2.05 |
| **18** | 1 | 325 | 0 | 132 | -2 | 0.1 | 1 | 9 | 1.5 | 1.37 |
| **19** | 1 | 325 | 1 | 186 | -1 | 0.18 | 2 | 11 | 1.73 | 1.19 |
| **20** | 1 | 325 | 2 | 240 | 0 | 0.26 | -2 | 3 | 2.15 | 2.18 |
| **21** | 2 | 410 | -2 | 24 | 2 | 0.42 | 1 | 9 | 2.72 | 2.28 |
| **22** | 2 | 410 | -1 | 78 | -2 | 0.1 | 2 | 11 | 1.16 | 1.48 |
| **23** | 2 | 410 | 0 | 132 | -1 | 0.18 | -2 | 3 | 2.5 | 2.20 |
| **24** | 2 | 410 | 1 | 186 | 0 | 0.26 | -1 | 5 | 2.3 | 2.45 |
| **25** | 2 | 410 | 2 | 240 | 1 | 0.34 | 0 | 7 | 2.2 | 2.46 |

**Table 5.9 Prediction error (%) of RSM**

| Exp No. | Speed, rpm | | Force, N | | Feed, mm/rev | | Ball Diameter | | Responses | | Prediction Error (%) |
|---|---|---|---|---|---|---|---|---|---|---|---|
| | Coded Value | Actual Value | Coded Value | Actual Value | Coded Value | Actual Value | Coded Value | Actual Value | Ra, µm | Ra (RSM Model) | |
| **4** | -2 | 70 | 1 | 186 | 1 | 0.34 | 1 | 9 | 2.26 | 2.14 | -5.30% |
| **8** | -1 | 115 | 0 | 132 | 1 | 0.34 | 2 | 11 | 1.22 | 1.22 | 0% |
| **9** | -1 | 115 | 1 | 186 | 2 | 0.42 | -2 | 3 | 2.12 | 2.17 | 2.35% |
| **11** | 0 | 240 | -2 | 24 | 0 | 0.26 | 2 | 11 | 1.23 | 1.28 | 4.06% |
| **15** | 0 | 240 | 2 | 240 | -1 | 0.18 | 1 | 9 | 1.72 | 1.69 | -1.74% |
| **16** | 1 | 325 | -2 | 24 | 1 | 0.34 | -1 | 5 | 2.55 | 2.41 | -5.49% |
| **18** | 1 | 325 | 0 | 132 | -2 | 0.1 | 1 | 9 | 1.5 | 1.37 | 8.67% |
| **20** | 1 | 325 | 2 | 240 | 0 | 0.26 | -2 | 3 | 2.15 | 2.18 | 1.39% |
| **24** | 2 | 410 | 1 | 186 | 0 | 0.26 | -1 | 5 | 2.3 | 2.45 | 6.52% |

The Prediction error of Experiment 4, 8, 9,11,15,16,18,20,24 is less than ±10% So we consider the model as a valid model [35, 36].

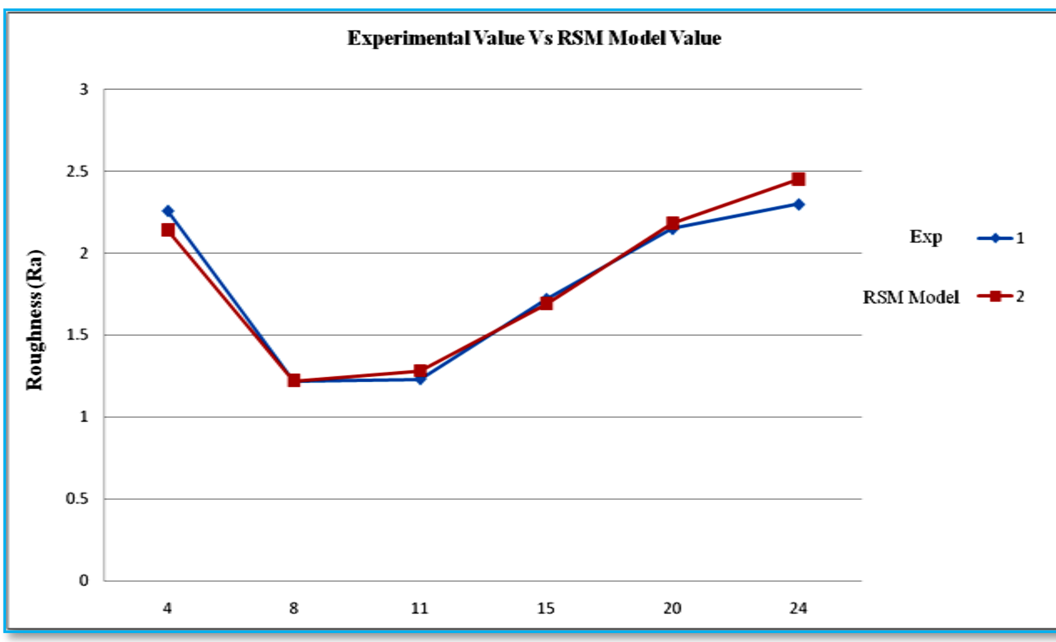


Fig 5.2 Experimental value Vs RSM model value

Graph in fig 5.2 compares the experimental response and RSM model response. It can be seen from the graph that the responses from the experiment and RSM model are almost same 4, 8,11,15,20 and 24 experiments. From the experiment 4 to 8 the roughness gradually decreases and remains steady from the experiment 8 to 11. Beyond this portion up to experiment 24 the roughness increases again. It can be concluded that the best set burnishing parameters would be exist in the experiments 8 to 11.

### 5.4.2 Effect of burnishing parameters on roughness according to RSM

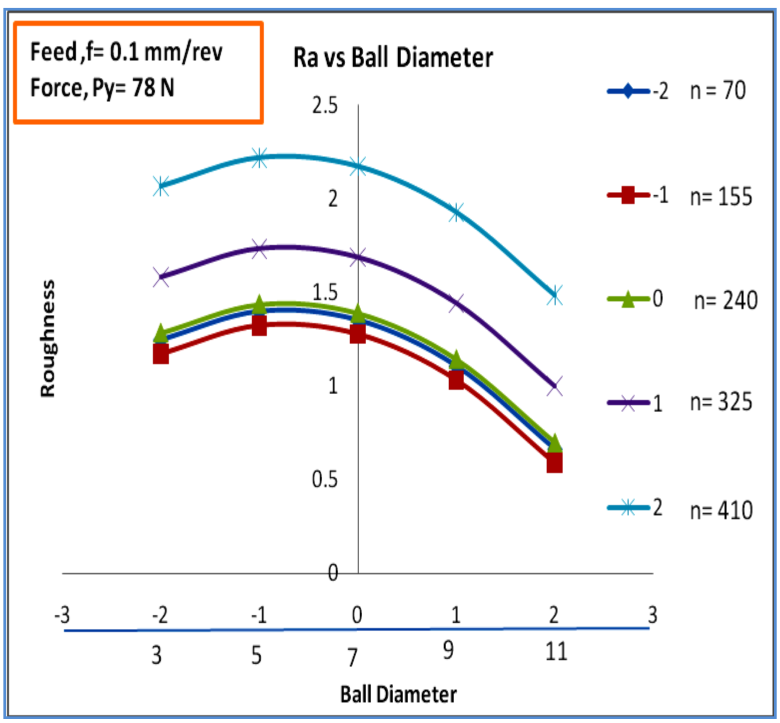


Fig. 5.3 Effect of ball diameter on roughness at different speed

**Ball diameter**

Figure 5.3 shows the effect of ball diameter on the roughness for a different speed. From 3mm to 5 mm, roughness increases with the increases of ball diameter. After 5 mm, roughness decreases with the increases in ball diameter and it remains up to

11mm. The results indicate that the roughness is low at ball diameter 11mm at 155 rpm.

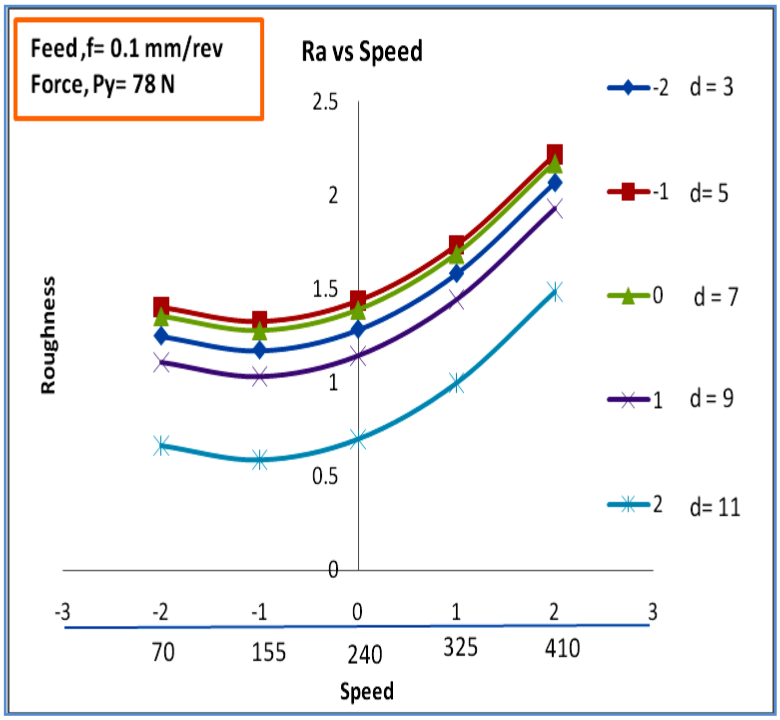


Fig. 5.4 Effect of speed on roughness at different ball diameter

**Burnishing Speed**

The effects of burnishing speed on the roughness for various ball diameters are shown in Fig. 5.4. It can be observed in Fig. 5.4 that the relationship between the roughness and the burnishing speed is parabolic. For each graph, the roughness goes though a minimum value at a given burnishing speed. Examining all ball diameters, the value of burnishing speed at which a minimum roughness is achieved ranges from 70 to 155 rpm. After 155 rpm, roughness increases with the increase of speed. It is also indicated that at speed 155 rpm and ball diameter 11 values of roughness is achieved at low.

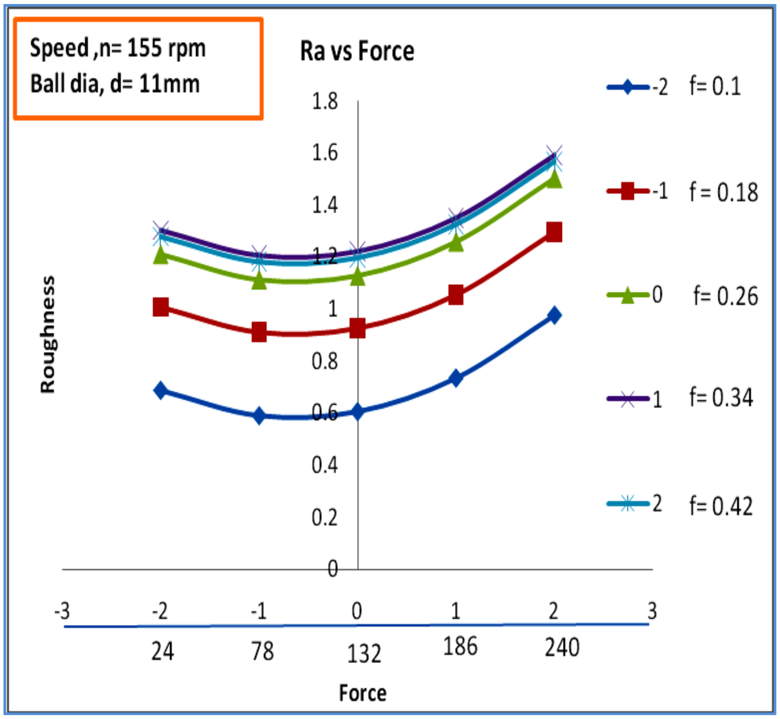


Fig. 5.5 Effect of force on roughness at different feed

**Burnishing Force**

Figure 5.5 presents the variations of the roughness with burnishing force at the different feed, indicating that the roughness can be significantly altered by the burnishing force. According to Fig. 5.5, the minimum roughness is obtained with a burnishing feed of 0.1mm/rev. Therefore, a feed of 0.1 mm/rev is considered the optimum burnishing feed for MS shaft. It should be noted that at a feed of 0.1 mm/rev, a combination of burnishing force 78 N is detrimental to the roughness. This is because high forces cause shear failure in the subsurface layers which in turn causes flaking.

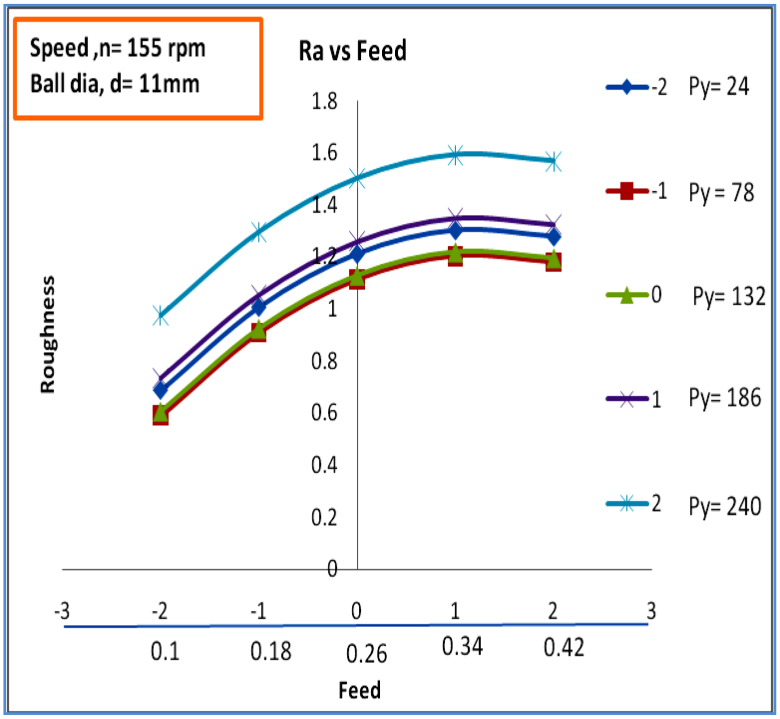


Fig. 5.6 Effect of feed on roughness at different force

**Burnishing Feed**

Figure 5.6 shows the effect of feed rate on the roughness for different burnishing force and at a fixed speed (n = 155 rpm) and ball diameter (d = 11 mm). The results indicate that roughness is proportioned to feed rate and it increases with the feed. The force 78 N curve shows the lowest roughness at the different feed. The combination of 78 N force and 0.1 mm/rev is given the desired roughness.

## 5.5 Formulation of Generalized Experimental Data Base Model

The relation for Ra may be expressed as

$R_a = f\,(n, P_{Y,}\ f, d, H_B)$

Which is most general form may be written as

$f\,(R_a, n, P_{Y,}\ f, d, H_B) = C$

Thus in present case, the number of variables $n$ = 6 and these can be described by three fundamental dimensions. Hence $m$ = 3 and ($n$- $m$) = 3. Thus there are 3 dimensionless π- groups in terms of which the above functional relationship may be expressed.

Choosing n, d, $H_B$ as repeating variables, we have

$\pi 1 = n^{a1}\ H_B^{\ b1}\ d^{c1}\ R_a$

$\pi 2 = n^{a2}\ H_B^{\ b2}\ d^{c2}\ f$

$\pi 3 = n^{a3}\ H_B^{\ b3}\ d^{c3}\ P_Y$

Now,

$\pi 1 = n^{a1}\ H_B^{\ b1}\ d^{c1}\ R_a$

$$[M^0L^0T^0] = [M^0L^0T^{-1}]^{a1}\ [M^1L^{-1}T^{-2}]^{b1}\ [M^0L^1T^0]^{c1}[M^0L^1T^0]$$

Equating the exponents of M, L and T we get

For M: $b_1 = 0$

For L: $0 = -\ b_1 + c_1 + 1$

For T: $0 = -\ a_1 - 2b_1$

From which $a_1 = 0,\ b_1 = 0,\ c_1 = -1$

$\pi 1 = n^0\ H_B^{\ 0}\ d^{-1}\ R_a$

$\pi 1 = (R_a/d)$

Similarly, we have

$\pi2 = n^{-1} H_B^{0} d^{-1} f$

$\pi2 = (f/nd)$

$\pi3 = n^{0} H_B^{-1} d^{-2} P_Y$

$\pi3 = (P_Y/H_B d^2)$

Thus it obtained

$(R_a/d) = f\,[(f/nd), (P_Y/H_B d^2)]$

$R_a = K\,(d) \times [(f/nd) \times (P_Y/H_B d^2)]$

The value of π1, π2 and π3 is shown as **Appendix-2**

**Table 5.10 Logarithmic Value of π2, π3, πRa**

| Exp No. | Log(π2) | Log(π3) | Log(π2²) | Log(π3²) | Log(π2xπ3) | LogπRa |
|---|---|---|---|---|---|---|
| **1** | -3.32239 | -2.37675 | -6.64479 | -4.7535 | -5.69914 | -0.27572 |
| **2** | -3.28904 | -2.3098 | -6.57807 | -4.61961 | -5.59884 | -0.63827 |
| **3** | -3.27572 | -2.37675 | -6.55145 | -4.7535 | -5.65247 | -0.42022 |
| **4** | -3.26841 | -2.4437 | -6.53682 | -4.88739 | -5.71211 | -0.60206 |
| **5** | -3.2636 | -2.50864 | -6.52721 | -5.01728 | -5.77224 | -0.79588 |
| **6** | -3.78252 | -3.11351 | -7.56503 | -6.22702 | -6.89603 | -0.49485 |
| **7** | -3.73049 | -2.82391 | -7.46097 | -5.64782 | -6.5544 | -0.88606 |
| **8** | -3.70115 | -2.76955 | -7.40229 | -5.5391 | -6.4707 | -0.95861 |
| **9** | -3.04431 | -1.48812 | -6.08862 | -2.97623 | -4.53243 | -0.14874 |
| **10** | -3.88941 | -1.82391 | -7.77882 | -3.64782 | -5.71332 | -0.50864 |
| **11** | -4.00877 | -3.50864 | -8.01755 | -7.01728 | -7.51741 | -0.95861 |
| **12** | -3.32606 | -1.86646 | -6.65212 | -3.73292 | -5.19252 | -0.3098 |
| **13** | -3.45593 | -2.08092 | -6.91186 | -4.16184 | -5.53685 | -0.22915 |
| **14** | -4.22185 | -2.22185 | -8.4437 | -4.4437 | -6.4437 | -0.76955 |
| **15** | -4.08092 | -2.3279 | -8.16184 | -4.6558 | -6.40882 | -0.72125 |
| **16** | -3.67985 | -2.82391 | -7.35971 | -5.64782 | -6.50376 | -0.29243 |
| **17** | -3.73283 | -2.60206 | -7.46566 | -5.20412 | -6.33489 | -0.74473 |
| **18** | -4.46852 | -2.58503 | -8.93704 | -5.17005 | -7.05355 | -0.76955 |
| **19** | -4.30103 | -2.61979 | -8.60206 | -5.23958 | -6.92082 | -0.79588 |
| **20** | -3.57349 | -1.37675 | -7.14698 | -2.7535 | -4.95024 | -0.14267 |
| **21** | -3.9431 | -3.3279 | -7.88619 | -6.6558 | -7.271 | -0.52288 |
| **22** | -3.65561 | -3 | -7.31122 | -6 | -6.65561 | -1 |
| **23** | -3.83565 | -1.63827 | -7.67129 | -3.27654 | -5.47392 | -0.08092 |
| **24** | -3.8962 | -1.95861 | -7.79239 | -3.91721 | -5.8548 | -0.33724 |
| **25** | -3.92812 | -2.14874 | -7.85624 | -4.29748 | -6.07686 | -0.50864 |

The relationship between various parameters was unknown. The dependent parameter $\pi Ra$ bear an intricate relationship with $\pi 2$, $\pi 3$ evaluated on the basis of experimentation. The true relationship is difficult to obtain. The possible relation may be linear, log-linear, polynomial with n degrees, linear with products of independent $\pi i$ terms. In this manner, any complicated relationship can be evaluated and further investigated for error. Hence the relationship for $R_a$ has formulated as:

$$\pi Ra = k_1 \times (\pi 2)^{a1} \times (\pi 3)^{b1}$$

Equation is modified as:

Obtaining Log on both sides we get,

$$\text{Log}(\pi Ra) = \text{Log}(k_1) + a_1 \text{Log}(\pi 2) + b_1 \text{Log}(\pi 3)$$

This linear relationship now can be viewed as the hyperplane in seven-dimensional spaces. To simplify further let us replace log terms by capital alphabet terms implies,

Let,

$Z_1 = \text{Log}(\pi Ra)$, $K_1 = \text{Log}(k_1)$, $A = \text{Log}(\pi 2)$, $B = \text{Log}(\pi 3)$

Putting the values in equations, the same can be written as

$$Z_1 = K_1 + a_1 A + b_1 B$$

Applying the theories of regression analysis on Table 5.11, the aim is to minimize the error $(E) = Y_e - Y_c$. $Y_c$ is the computed value of $\pi Ra$ using regression equation and $Y_e$ are the value of the same term obtained from experimental data with exactly the same values of $\pi 2$ to $\pi 3$. Correlation and reliability were computed for model accuracy.

Minitab16 Software Regression analysis:

**Regression Analysis: LogπRa versus Log(π2xπ3)**

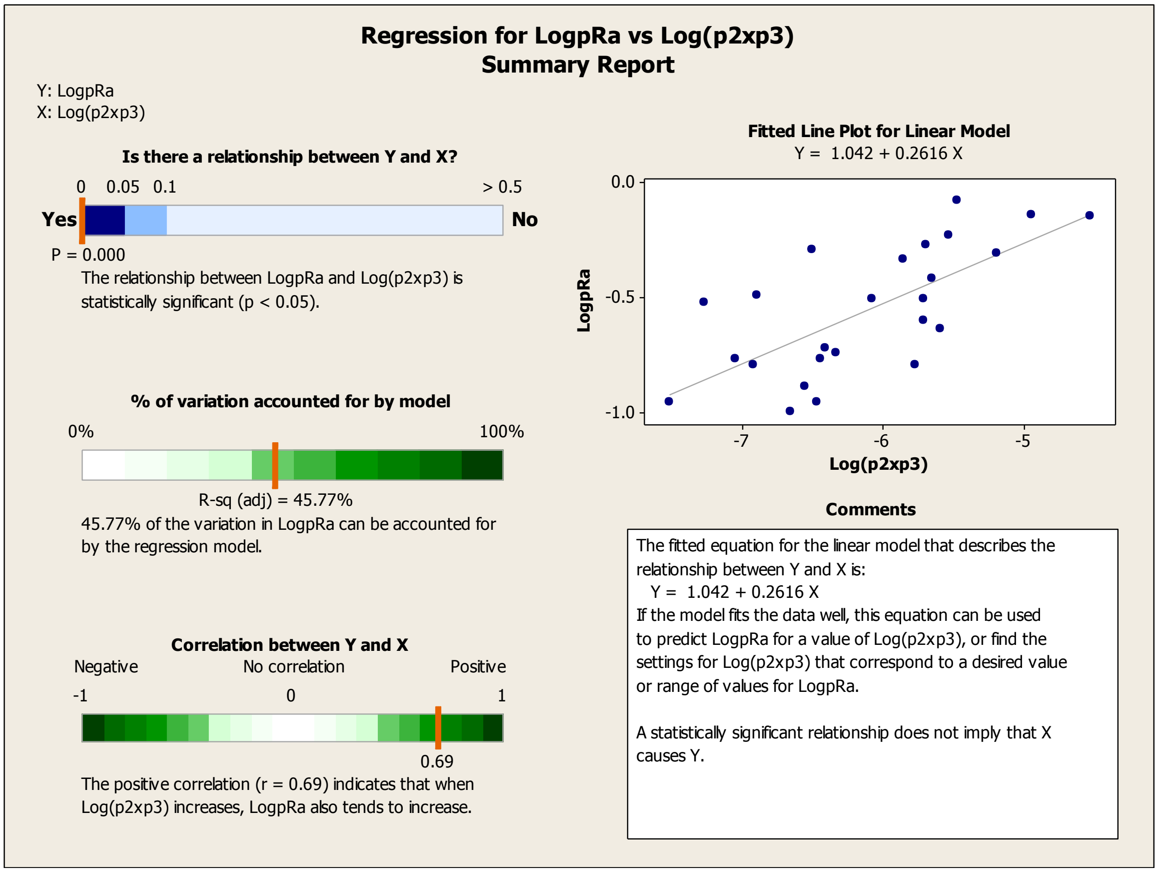


Fig. 5.7 Regression Analysis of LogπRa versus Log(π2xπ3)

The regression equation is

$$\text{Log}(\pi Ra) = 1.042 + 0.2616\ \text{Log}(\pi 2 x \pi 3)$$

$$\text{Log}(\pi Ra) = \text{Log}(11.015) + 0.2616\ \text{Log}(\pi 2 x \pi 3)$$

$$\pi Ra = 11.015 \times (\pi 2 x \pi 3)^{0.2616}$$

It is necessary to correlate quantitatively various independent and dependent terms involved in this very complex phenomenon. This correlation is nothing but a mathematical model as a design tool for such situation. The mathematical model for internal knot removal operation is shown below:

$$(R_a/d) = 11.015 \times [(f\ P_Y/nH_Bd^3)^{0.2616}]$$

$$R_a = 11.015 \times (f\ P_Y /n\ H_B)^{0.2616} \times d^{0.215}$$

### 5.5.1 Validation of Mathematical Model

The purpose of the validation is to evaluate the accuracy of the prediction model with the experimental data**.** In this work, the prediction errors are defined as equation 19.

$$\text{Prediction error} = \frac{\textbf{Pred. result} - \textbf{Exp. result}}{\textbf{Exp. result}} \times 100\%$$

**Table 5.11 Prediction error (%) of Mathematical model**

| Exp No. | π2 | π3 | π2x π3 | πRa= $R_a$/d (Model) | Ra (Model) | Ra (Exp) | Prediction Error (%) |
|---|---|---|---|---|---|---|---|
| **3** | 0.00053 | 0.0042 | 2.226E-06 | 0.37 | 2.59 | 2.68 | -3.35% |
| **9** | 0.000903 | 0.0325 | 2.9348E-05 | 0.72 | 2.16 | 2.12 | 1.88% |
| **10** | 0.000129 | 0.015 | 1.935E-06 | 0.35 | 1.70 | 1.55 | 9.67% |
| **11** | 0.000098 | 0.00031 | 3.038E-08 | 0.12 | 1.32 | 1.23 | 7.31% |
| **12** | 0.000472 | 0.0136 | 6.4192E-06 | 0.48 | 1.44 | 1.47 | -2.04% |
| **18** | 0.000034 | 0.0026 | 8.84E-08 | 0.16 | 1.44 | 1.5 | -4% |
| **19** | 0.00005 | 0.0024 | 0.00000012 | 0.17 | 1.87 | 1.73 | 8.09% |
| **25** | 0.000118 | 0.0071 | 8.378E-07 | 0.28 | 1.98 | 2.2 | -10% |

The Prediction error of Experiment 3, 9,10,11,12,18,19,25 is less than ±10%
So the model is valid [35,36].

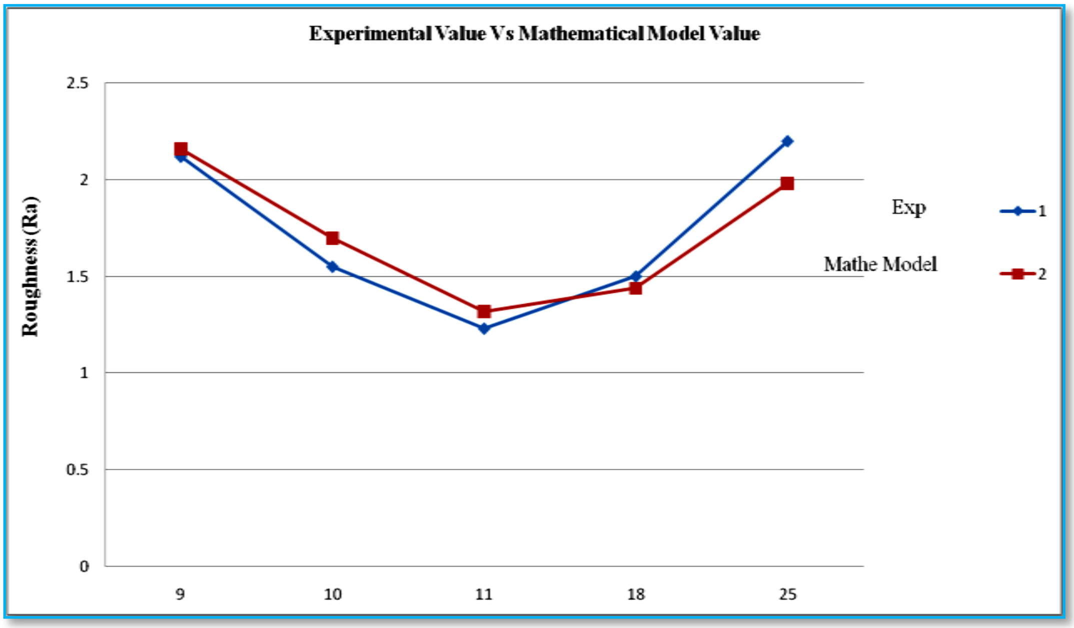


Fig. 5.8 Experimental Value (Ra) Vs Mathematical Value (Ra)

Graph in fig 5.8 compares the experimental response to data base mathematical model response. It can be seen from the graph that the responses characteristic from the experiment and mathematical model are almost same at 9, 10,11,18 and 25 no. experiments. From the experiment 9 to 11 the roughness gradually decreases and it is clear that at experiment 11exist lowest roughness. Beyond this portion, from experiment 11 to 25 the roughness increases again. It can be concluded that the nature of surface rough curve with respect to burnishing parameters is parabolic.

### 5.6 Comparison of Experimental Value, RSM and Mathematical Model Value

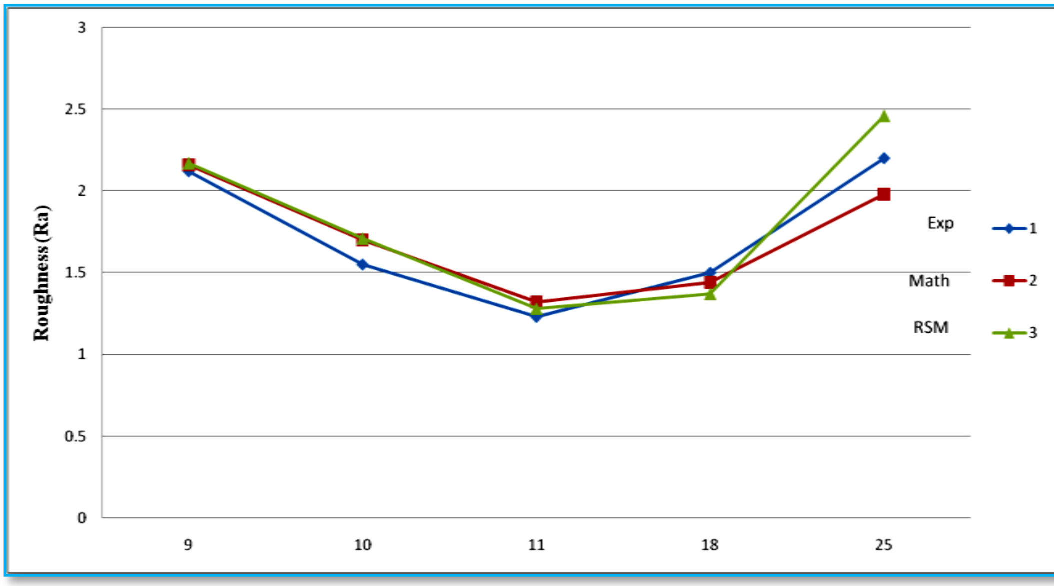


Fig. 5.9 Comparison of Experimental Value, RSM and Mathematical Model Value

Graph in fig 5.9 compares the experimental, RSM and mathematical model responses. It can be seen from the graphs that the responses characteristic from the experiment, RSM and mathematical model are almost same at 9, 10, 11, 18 and 25 no. experiments. From the experiment 9 to 11 the roughness gradually decreases and it is clear that at experiment 11exist lowest roughness. Beyond this portion, from experiment 11 to 25 the roughness increases again. It can be concluded that the nature of surface rough curve with respect to burnishing parameters is parabolic.

### 5.7 Discussions on result

The following points can be drawn based on the above experimental results of this study

- From Taguchi method, we get the optimum surface roughness at ball diameter 11mm.To prove this result Response Surface methodology (RSM) has been applied and it shows the same ball diameter for optimum surface roughness. Initially, roughness increases with the increases of ball diameter. After ball diameter 5 mm, roughness decreases with the increases in ball diameter and it remains up to 11mm. The roughness is low at ball diameter 11mm.

- The relationship between the roughness and the burnishing speed is parabolic. The roughness goes though a minimum value at a given burnishing speed. The minimum roughness is achieved ranges from 70 to 155 rpm. After 155 rpm, roughness increases with the increase of speed. From RSM and Taguchi method at speed 155 rpm the low roughness is achieved.

- The roughness can be significantly altered by the burnishing force. At lower and higher force roughness is high. From RSM the lower surface roughness is achieved at 78 N. After 78 N force, the roughness is directly proportional to roughness. The high forces cause shear failure in the subsurface layers which in turn causes flaking.

- It is clear from RSM that the roughness is proportioned to feed rate and it increases with the feed. At higher feed rate gives higher surface roughness. The feed rate 0.1 mm/rev is given the desired roughness in this study. By applying Taguchi Method the same result is achieved.

- Formulation of generalized experimental database model by dimensional analysis. The mathematical model for roughness
  $R_a = 11.015 \text{ x } (f\, P_Y /n\, H_B)^{0.2616} \text{ x } d^{0.215}$

- By ANOVA, the percentage of contribution of the individual parameter is evaluated. The contributions of speed, force, feed and ball diameter are 10.73%, 34.00%, 19.61% and 17.00% respectively. It is clear that force has a great impact on surface roughness.

# CHAPTER-6

## CONCLUSIONS AND RECOMMENDATION

The effect of ball burnishing speed, feed, force and ball diameter on the surface roughness of Mild Steels were studied for a special designed burnishing tool. The main results obtained are as follows:

- In this research a flexible ball burnishing tool is designed with possibility use of different ball diameter fast replacing it.

- In this study the relationship among the surface roughness and the burnishing parameter speed, feed, force and ball diameter are established.

- Optimum burnishing parameters for the given materials (Mild Steel) using the designed tool is established by Taguchi L25 matrix and these can be used for the selection of the parameter for given surface roughness.
- Experimental results show that the highest surface finish effect can be achieved at feed of 0.1 mm/rev, speed of 155 rpm, burnishing force of 78N and ball diameter of 11 mm.

- Experimental work shows that an improvement of about 60% in the surface roughness of Mild Steel can be obtained by using the designed tool.

# CHAPTER-7

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# Appendix-1

## Regression Analysis: Ra versus X1, X2, X3, X4

```
* X2xX3 is highly correlated with other X variables
* X2xX3 has been removed from the equation.


The regression equation is
Ra = 1.86 + 0.0695 X1 + 0.0273 X2 + 0.136 X3 - 0.179 X4 + 0.0936 X1xX1
     + 0.0567 X2xX2 - 0.0573 X3xX3 - 0.0992 X4xX4 - 0.0452 X1xX2 - 0.045 X1xX3
     - 0.0166 X3xX4


Predictor      Coef  SE Coef      T      P
Constant     1.8646   0.3318   5.62  0.000
X1          0.06950  0.08911   0.78  0.449
X2          0.02726  0.09593   0.08  0.941
X3          0.13598  0.08911   1.53  0.151
X4         -0.17876  0.09400  -1.90  0.080
X1xX1       0.09358  0.07851   1.19  0.255
X2xX2       0.05665  0.08308   0.68  0.507
X3xX3      -0.05734  0.07851  -0.73  0.478
X4xX4      -0.09920  0.08464  -1.17  0.262
X1xX2      -0.04525  0.08308  -0.54  0.595
X1xX3       -0.0451   0.1093  -0.41  0.687
X3xX4      -0.01660  0.08308  -0.20  0.845


S = 0.557434   R-Sq = 48.8%   R-Sq(adj) = 5.4%


Analysis of Variance


Source          DF      SS      MS     F      P
Regression      11  3.8447  0.3495  1.12  0.415
Residual Error  13  4.0395  0.3107
Total           24  7.8842


Source  DF  Seq SS
X1       1  0.3026
X2       1  0.0444
X3       1  1.2577
X4       1  0.8924
X1xX1    1  0.5092
X2xX2    1  0.0510
X3xX3    1  0.0844
X4xX4    1  0.3774
X1xX2    1  0.2128
X1xX3    1  0.1003
X3xX4    1  0.0124


Unusual Observations

Obs    X1     Ra    Fit  SE Fit  Residual  St Resid
 13  0.00  2.950  2.020   0.339     0.930     2.10R

R denotes an observation with a large standardized residual.
```

# Appendix-2

The Value of $\pi1$, $\pi2$ and $\pi3$

| Exp No. | $\pi2$ | $\pi3$ | $\pi2^2$ | $\pi3^2$ | $\pi2$x $\pi3$ | $\pi1=\pi Ra= R_a/d$ |
|---|---|---|---|---|---|---|
| **1** | 0.000476 | 0.0042 | 2.26576E-07 | 0.00001764 | 1.9992E-06 | 0.53 |
| **2** | 0.000514 | 0.0049 | 2.64196E-07 | 0.00002401 | 2.5186E-06 | 0.23 |
| **3** | 0.00053 | 0.0042 | 2.809E-07 | 0.00001764 | 2.226E-06 | 0.38 |
| **4** | 0.000539 | 0.0036 | 2.90521E-07 | 0.00001296 | 1.9404E-06 | 0.25 |
| **5** | 0.000545 | 0.0031 | 2.97025E-07 | 0.00000961 | 1.6895E-06 | 0.16 |
| **6** | 0.000165 | 0.00077 | 2.7225E-08 | 5.929E-07 | 1.2705E-07 | 0.32 |
| **7** | 0.000186 | 0.0015 | 3.4596E-08 | 0.00000225 | 2.79E-07 | 0.13 |
| **8** | 0.000199 | 0.0017 | 3.9601E-08 | 0.00000289 | 3.383E-07 | 0.11 |
| **9** | 0.000903 | 0.0325 | 8.15409E-07 | 0.00105625 | 2.9348E-05 | 0.71 |
| **10** | 0.000129 | 0.015 | 1.6641E-08 | 0.000225 | 1.935E-06 | 0.31 |
| **11** | 0.000098 | 0.00031 | 9.604E-09 | 9.61E-08 | 3.038E-08 | 0.11 |
| **12** | 0.000472 | 0.0136 | 2.22784E-07 | 0.00018496 | 6.4192E-06 | 0.49 |
| **13** | 0.00035 | 0.0083 | 1.225E-07 | 0.00006889 | 2.905E-06 | 0.59 |
| **14** | 0.00006 | 0.006 | 3.6E-09 | 0.000036 | 0.00000036 | 0.17 |
| **15** | 0.000083 | 0.0047 | 6.889E-09 | 0.00002209 | 3.901E-07 | 0.19 |
| **16** | 0.000209 | 0.0015 | 4.3681E-08 | 0.00000225 | 3.135E-07 | 0.51 |
| **17** | 0.000185 | 0.0025 | 3.4225E-08 | 0.00000625 | 4.625E-07 | 0.18 |
| **18** | 0.000034 | 0.0026 | 1.156E-09 | 0.00000676 | 8.84E-08 | 0.17 |
| **19** | 0.00005 | 0.0024 | 2.5E-09 | 0.00000576 | 0.00000012 | 0.16 |
| **20** | 0.000267 | 0.042 | 7.1289E-08 | 0.001764 | 1.1214E-05 | 0.72 |
| **21** | 0.000114 | 0.00047 | 1.2996E-08 | 2.209E-07 | 5.358E-08 | 0.30 |
| **22** | 0.000221 | 0.001 | 4.8841E-08 | 0.000001 | 2.21E-07 | 0.1 |
| **23** | 0.000146 | 0.023 | 2.1316E-08 | 0.000529 | 3.358E-06 | 0.83 |
| **24** | 0.000127 | 0.011 | 1.6129E-08 | 0.000121 | 1.397E-06 | 0.46 |
| **25** | 0.000118 | 0.0071 | 1.3924E-08 | 0.00005041 | 8.378E-07 | 0.31 |