



# **A Numerical Study of Vertical Discontinuity of RCC Frame Structures**

A Thesis Submitted in Partial Fulfillment of the Requirements for the  
Bachelor of Science Degree in Civil Engineering.

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# DECLARATION

We hereby declare that the undergraduate project work reported in this thesis has been performed by us under the supervision of Dr. Md. Jahidul Islam and this work has not been submitted elsewhere for any purpose (except for publication).

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*Dedicated*

*To*

*Our Parents*

## ACKNOWLEDGEMENT

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## ABSTRACT

Irregular Structures are a common phenomenon in modern day urbanization. Structures having significant physical discontinuities in vertical configuration or in their lateral force resisting systems are termed as vertically irregular structure. Irregularity is an inevitable occurrence in RCC frame structures now days. The effect of this irregularity can be devastating. The software based analysis that is called ABAQUS 6.10 is used for finite element analysis through simulating stress analysis on irregular structure.

Keywords: irregularity, discontinuity, stress analysis, finite element analysis, ABAQUS

## Contents

DECLARATION .....	i
ACKNOWLEDGEMENT .....	iii
ABSTRACT.....	iv
LIST OF FIGURES & TABLES .....	vii
CHAPTER ONE .....	1
1.1 General.....	2
1.2 Background .....	2
1.3 Vertical Discontinuity in building structures.....	7
1.4. Soft story structure.....	9
1.5 Literature Review .....	10
1.6 Overview from Literature Review.....	17
1.7 Objectives and Scope of the Study .....	18
1.8 Organization of Thesis.....	18
CHAPTER TWO .....	19
2.1. General.....	20
2.2 Finite Element Analysis.....	20
2.3 Material Modelling .....	21
2.3.1 Concrete Damage Plasticity.....	21
2.3.2 Linear Elastic Material Model .....	23
2.4 Embedded Element .....	24
2.5 Boundary Condition.....	25
2.6 Loading Conditions .....	26
2.7 Finite Element Software .....	26
2.8 Selection of ABAQUS to Perform Finite Element Analysis.....	26
2.9 Load Calculation.....	27
CHAPTER THREE .....	29
3.1 General.....	30
3.2 Pre Processing.....	30
3.2.1 Frame Modelling .....	30
3.2.2 Reinforcement Modelling.....	30
3.3 Case 1.....	32
3.4 Case 2.....	32
3.4 Material Properties of Concrete Beam .....	33

3.5 Material Properties of Reinforcement bar .....	34
CHAPTER FOUR .....	35
4.1 General.....	36
4.2 Comparative Study of Von Mises Stress .....	36
4.2.1 Case 1.....	36
4.2.2 Case 2.....	37
4.3 Comparative Study of Tresca Stress.....	37
4.3.1 Case 1.....	37
4.3.2 Case 2.....	38
4.4 Comparative Study of Maximum Principle Stress .....	39
4.4.1 Case 1.....	39
4.4.2 Case 2.....	40
CHAPTER FIVE .....	41
5.1 General.....	42
5.2 Comparison of maximum and minimum values of different stresses .....	42
5.3 Recommendation and Conclusion .....	43
REFERENCES .....	44

# LIST OF FIGURES & TABLES

FIGURE 1.1	COLLAPSE OF STRUCTURE DUE TO EARTHQUAKE .....	3
FIGURE 1.2	SOFT-STORY MECHANISM IN THE GROUND FLOOR IN A COMMERCIAL BUILDING DURING 1997 MANAGUA EARTHQUAKE IN NICARAGUA.. .....	4
FIGURE 1.3	OLIVE VIEW HOSPITAL, SAN FERNANDO, CALIFORNIA. PARTIAL VIEW OF THE 5-STORY MEDICAL TREATMENT AND CARE.....	5
FIGURE 1.4	DAMAGE TO COLUMNS IN HIMGIRI APARTMENT.....	6
FIGURE 1.5	DAMAGE TO COLUMNS IN THE STILT STOREY OF YOUTH HOSTEL BUILDING.....	6
FIGURE 1.6	STIFFNESS IRREGULARITY.....	7
FIGURE 1.7	WEIGHT (MASS) IRREGULARITY.....	7
FIGURE 1.8	VERTICAL GEOMETRIC IRREGULARITY.....	8
FIGURE 1.9	IN-PLANE DISCONTINUITY.....	8
FIGURE 1.10	DISCONTINUITY IN CAPACITY.....	9
FIGURE 1.11	SOFT-STOREY STRUCTURE.....	9
FIGURE 2.1	FINITE ELEMENT EQUATION.....	20
FIGURE 2.2	FINITE ELEMENT ANALYSIS.....	21
FIGURE 2.3	CONCRETE DAMAGE PLASTICITY .....	22
FIGURE 2.4	LINEAR ELASTIC MODEL.....	23
FIGURE 2.5	EMBEDDED ELEMENT .....	25
FIGURE 3.1	CONCRETE FRAME MODELLING .....	30
FIGURE 3.2	BEAM REINFORCEMENT.....	31
FIGURE 3.3	COULUMN REINFORCEMENT .....	32
FIGURE 3.4	APPLICATION OF HORIZONTAL AND VERTICAL LOAD ON REGULAR FRAME STRUCTURE .....	32
FIGURE 3.5	APPLICATION OF HORIZONTAL AND VERTICAL LOAD ON IRREGULAR FRAME STRUCTURE .....	33
FIGURE 4.1	VON MISES STRESS DISTRIBUTION IN REGULAR STRUCTURE	36
FIGURE 4.2	VON MISES STRESS DISTRIBUTION IN IRREGULAR STRUCTURE	37



FIGURE 4.3	TRESCA STRESS DISTRIBUTION IN REGULAR STRUCTURE	37
FIGURE 4.4	TRESCA STRESS DISTRIBUTION IN IRREGULAR STRUCTURE	38
FIGURE 4.5	MAXIMUM PRINCIPLE STRESS DISTRIBUTION IN REGULAR STRUCTURE	39
FIGURE 4.6	MAXIMUM PRINCIPLE STRESS DISTRIBUTION IN IRREGULAR STRUCTURE	40
TABLE 5.2	COMPARISON OF MAXIMUM AND MINIMUM VALUES OF DIFFERENT STRESSES.....	42

# CHAPTER ONE

## **INRODUCTION**

## 1.1 General

In the recent practice of structural engineering, we can see that different kinds of structures are being built around the world; such as: Soft Story Structures, Irregular Structures, and Setback Structures etc. In our topic, we want to focus on structures where the space is narrow and want to provide vertical discontinuity. Stress analysis is done through ABAQUS software. Results of finite element analysis indicates the viability of the structure under irregularity and installation of discontinuous columns.

Proper understanding is required in terms of irregular behavior of vertical discontinuity of the structure. Irregularity in structures are defined in plan and in elevation. One of the irregularities in elevation is discontinuity in columns and shear walls. Discontinuities in shear walls are not permitted due to large bending moment and shear force to be transferred to the lower stories by means of the supporting beam. On the other hand shear walls supported by the columns are not permitted as well. Regular and symmetrical structures exhibit more favorable and predictable seismic response characteristics than irregular structures. Hence design of irregular structures carry more uncertainties.

## 1.2 Background

During the January 26, 2001 earthquake, numerous mid- to high-rise residential buildings collapsed in the city of Ahmedabad leading to several hundred casualties and significant financial loss. The city of Ahmedabad lies about 300 km (400 km by road) east of the epicenter of the January 26 event and falls in the seismic Zone III (IS: 1893-1976) of India. The lateral design forces for this region are about 4 to 6% of total weight of the building, depending on the foundation type and soil conditions. Given that the horizontal accelerations recorded in Ahmedabad during the earthquake event are about 10% of gravity, the buildings may be expected to deform slightly into the inelastic range. However, the extent of damage observed was significantly more than expected in such a moderate seismic region. Following is a brief summary of the reasons that contributed to this unexpected damage in residential construction.



Figure 1.1 Collapse of structure due to earthquake

The typical residential construction in Ahmedabad consists of reinforced concrete moment resisting frame system. The frame at the ground floor is open while frames at the upper floors are filled with un-reinforced brick panels. This type of lateral load resisting system leads to what is commonly known as a “soft-story” system. Most buildings also have overhanging covered balconies at higher floors; the overhangs were observed to be about 5 feet. The columns at the ground floor may not align with the columns at the upper floors giving rise to vertical discontinuities in the lateral load resisting system. The above-described lateral load resisting system occurs because of two factors. First, the open ground floor is needed to provide car parking; the buildings are usually built on very small land lots with little room for open parking. Second, the Floor Surface Index (FSI) used by the local municipal corporation for residential construction permits the land developers to cover more area at upper floor than the ground floor. The FSI only counts the area of within the column footprints at the ground floor. Therefore the developers are tempted to design the lateral load resisting system with only two to three columns in a frame on the ground floor with a beam overhangs on both sides. The upper floors may or may not continue these columns. But at least two floating columns are added, one on each

end of the cantilever beam, starting from the first floor and running the entire height of the building. The most residential buildings appear to be designed primarily for gravity load; there are some indications that the lateral loads may not have been properly considered in design of these buildings. There is insufficient confining steel to provide required ductility in the lateral load resisting system, and column reinforcement is spliced just above the beam level, with often insufficient development length.



Figure 1.2 Soft-story mechanism in the ground floor in a commercial building during 1997 Managua Earthquake in Nicaragua.

As shown in Figure 1.2 the hinging at the top and bottom of the first story columns were evident at all locations (Das, 2000). This first story was a ‘soft story’ because, except for glass partitions all around, it was completely open, while the second story had walls and partitions that increased significantly the lateral stiffness of this second story relative to the first. The olive view medical center was a 5 story reinforced concrete structure. Figure 1.4 illustrates the damage that olive view hospital suffered during the 1971 San Fernando Earthquake. As shown in Figure 1.4 a large permanent lateral second floor level displacement of the main Treatment and Care Unit was found. This large inter-story drift, which induced significant non-structural and structural damage and which led to the demolishing of the building, was a consequence of the formation of a soft story at the first story level because of the existence of a reinforced concrete wall above the second floor level (Bertero, 1997).



Figure 1.3. Olive View Hospital, San Fernando, California. Partial View of the 5-story Medical Treatment and Care

The Jabalpur earthquake of 22 May 1997 also illustrated the handicap of Indian buildings with soft first storey. This earthquake, the first one in an urban neighborhood in India, provided an opportunity to assess the performance of engineered buildings in the country during ground shaking. The damage incurred by Himgiri and Ajanta apartments in the city of Jabalpur are very good examples of the inherent risk involved in the construction of buildings with soft first storey. Himgiri apartments is a RC frame building with open first storey on one side for parking, and brick infill walls on the other side. The infill portion of the building in the first storey is meant for shops or apartments. All the storeys on top have brick infill walls. The first storey columns in the parking area were badly damaged including spalling of concrete cover, snapping of lateral ties, buckling of longitudinal reinforcement bars and crushing of core concrete (Fig. 1). The columns on the other side had much lesser level of damage in them. There was only nominal damage in the upper storeys consisting of cracks in the filler walls. This is a clear case of columns damaged as a result of the “soft first storey”. The Ajanta apartment’s buildings are a set of almost identical four storey RC frame building located side -by-side. In each of these buildings, there are two apartments in each storey, excepting the first storey. One building has two apartments in the upper stories, but only one apartment in the first stories. The open space on the other side is meant for parking, and hence has no infilled wall panels. Whereas, only nominal damages were reported in the building with two apartments the first storey, the first storey columns on the open side in the other building

were very badly damaged. The damage consisted of buckling of longitudinal bars, snapping of ties, spalling of cover and crushing of core concrete.



Figure 1.4 Damage to columns in Himgiri apartment.

In a two-storied (plus stilt storey) C-shaped RC frame building (Youth hostel building) in Jabalpur, the damage to the columns in the stilt storey consisted of severe X-type cracking due to cyclic lateral shear (Fig. 2). Here also, the two stories above the stilt storey have brick infilled wall panels. This makes the upper stories very stiff as compared to the storey at the stilt level. There was no damage to the columns in the stories above. The “soft first storey” at the stilt level is clearly the primary reason for such a severe damage.



Figure 1.5 Damage to columns in the stilt storey of Youth Hostel building.

### 1.3 Vertical Discontinuity in building structures

**Stiffness Irregularity**—Soft Storey: is defined to exist when there is a story in which the lateral stiffness is less than 70% of that in the story above or less than 80% of the average stiffness of the three stories above.

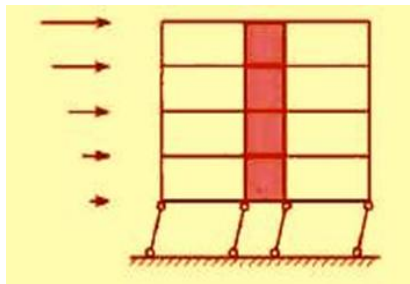


Figure 1.6: Stiffness Irregularity

**Stiffness Irregularity** -Extreme Soft Storey is defined to exist where there is a story in which the lateral stiffness is less than 60% of that in the story above or less than 70% of the average stiffness of the three stories above.

**Weight (Mass) Irregularity** is defined to exist where the effective mass of any story is more than 150% of the effective mass of an adjacent story. A roof that is lighter than the floor below need not be considered.

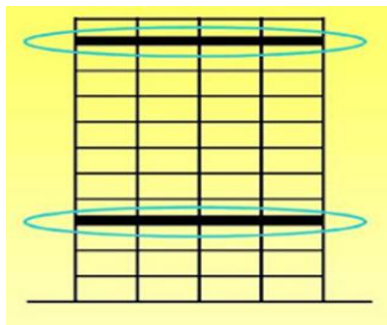


Figure 1.7: Weight (mass) irregularity



**Vertical geometric irregularity** shall be considered to exist where the horizontal dimension of the lateral force-resisting system in any story is more than 130% of that in an adjacent story.

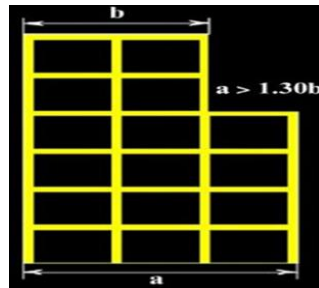


Figure 1.8: Vertical geometric irregularity

**In-plane Discontinuity** in Vertical Lateral-Force-Resisting Elements is defined to exist where an in plane offset of the lateral-force-resisting elements is greater than the length of those elements or where there is a reduction in stiffness of the resisting element in the story below.

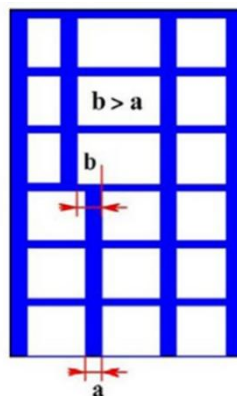


Figure 1.9: In-plane discontinuity

**Discontinuity in Capacity Weak Story** where the weak story is one in which the story lateral strength is less than 80% of that in the story above. The story lateral strength is the total lateral strength of all seismic-resisting elements sharing the story shear for the direction under consideration.

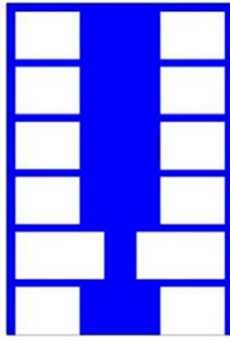


Figure 1.10 Discontinuity in Capacity

#### 1.4. Soft story structure

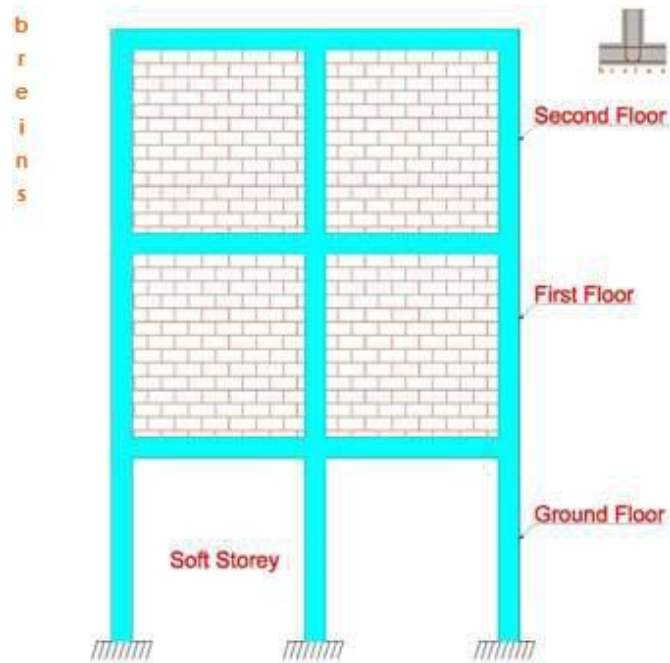


Figure 1.11 Soft storey structure

The lowest story in a building which qualifies as a story as defined herein, except that a floor level in a building having only one floor level shall be classified as a soft story, provided such floor level is not more than 1.25m below grade, as defined herein, for more than 50 percent of the total perimeter, nor more than 2.5m below grade at any point.

## 1.5 Literature Review

Humar and Wright (1977), using one ground motion record in their study, studied the dynamic behaviour of multi-storied steel rigid frame buildings with setback towers. They found that the difference in elastic and inelastic inter-story drifts between set-back and regular structures depends on the level of the story considered. For the tower, inter-story drifts were found to be larger than for regular structures. For the base, inter-story drifts were found to be smaller in set-back structures than in the regular ones. This observation agrees with the findings of Pekau and Green (1974).

Wood (1986) performed an experimental study on two small-scale set-back frames. She concluded that the behavior offset-back structures did not differ from the behavior of regular ones.

Aranda (1984) found that the ductility demands of columns and beams are higher for setback buildings than for regular ones. Aranda's study was performed using soft soil records from the 1980 Mexico earthquake. He concluded that the increase in ductility is more pronounced in the stories above the set-back level.

Sharooz and Moehle (1990) studied the effects of set-backs on the earthquake response on multi-story buildings. They observed, based on analytical studies, a concentration of damage in the tower due to high rotational ductility. They performed experiments on a set-back frame structure and concluded that the fundamental mode dominates the response in the direction parallel to the set-back, and that using static analysis should be sufficient to predict the response of set-back structures without the need to perform dynamic analysis.

Wong and Tsu (1994), studied the elastic response of setback structures by means of response spectrum analysis and found that the modal weights of higher order modes for setback structures are large, leading to a seismic load distribution that is different from static code procedures. They also found that for set-back structures, although higher order

modes may contribute more to the base shear than the fundamental mode, the first mode still dominates the displacement response.

Pinto and Costa (1995), studied Set-back structures and concluded that the seismic behavior of regular and irregular structures are similar. In their study the amount of discontinuity and the ratio of the base height to the total height were small.

Duan and Chandler (1995) pointed out that both static and modal spectral analyses were inadequate to prevent damage concentration in members near the setback level. This observation support the need for the development of new methods such as the DBD procedure proposed in this work.

Tena-Colunga (2004) studied two irregular (setback and slender) 14-storey RC moment resisting framed buildings, with one or two-bay frames in the slender direction. In this case, structures were designed close to the limiting drift angle of 1.2%, established by the Mexican code. Results obtained through nonlinear dynamic analyses suggested that the slender direction of setback buildings with one-bay frames is vulnerable, contrary to what occurs if a bay is added in the slender direction thanks to the higher redundancy in framed structures. The author concluded that seismic codes should penalize seismic design of buildings with single-bay frames in one direction.

Khoury et al. (2005) considered four 9-story asymmetric setback perimeter frame structures—designed according to the Israeli steel code SI 1225 (1998)—that differed with special attention on the influence of the setback level, nonlinear dynamic analyses were performed, and a 3D structural model was used under bi-directional ground motions. Results showed amplification in response at the upper tower stories, thus suggesting that the higher vibration modes have significant influence, particularly the torsional ones. In this respect, the authors recommended that future research on setback buildings should be conducted on full plan-asymmetric structures.

Athanassiadou and Bervanakis (2005) studied the seismic behavior of reinforced concrete buildings with setbacks designed to capacity design procedure provided by Euro code 8. In their study, two ten-story frames with two and four large setbacks in the upper floors respectively, as well as a third one, regular in elevation, have been designed to the provisions of Euro code 8 for the high (H) ductility class and a common peak ground

acceleration (PGA) of 0.25g. All frames were subjected to inelastic dynamic time-history analysis for selected input motions. They found that the seismic performance of the studied multistory reinforced concrete frame buildings with setbacks in the upper stories designed to EC8 can be considered as completely satisfactory, not inferior and in some cases even superior of that of the regular ones, even for motions twice as strong as the design earthquake. Inter-story drift ratios of irregular frames were found to remain quite low even in the case of the ‘collapse prevention’ earthquake with an intensity double that of the ‘design’ one.

Moehle and Sozen (1980), studied frame-wall structures possessing partial-height walls. Four 9-story model reinforced concrete structures were built, all possessing the same overall dimensions. To resist the seismic actions, in parallel to two full height frames, three of the structures used partial height walls of 1, 4 and 9 stories respectively, and the fourth had only the two frames to resist the seismic response without the walls. They found that the variations of top displacements with time of the structures with four and nine-story walls were nearly identical. The base shears that developed in the walls for both of these structures was approximately 60% of the total base shear. For the structure with a single story wall, the base shear in the wall was approximately 95% of the total base shear. Drifts were considerably greater in the lower stories of the single-story wall and pure frame structures. Due to the sharp change in story shear stiffness it might have been anticipated that the use of partial height walls would cause large shear demands around the point of wall termination. However the study showed that because the deformations of walls are primarily flexural, large story drifts could develop at intermediate stories (around the points of wall termination) without the development of large shears in the wall and frames. This point, together with the observation that top displacements of the structure with a full height wall were nearly identical to those of the structure with a four story wall, indicate that the use of partial height walls may be an acceptable frame-wall structural configuration.

Moehle (1984) studied the seismic response of four irregular reinforced concrete test structures. These test structures were simplified models of 9-story 3 bay building frames plane of these structures were introduced by discontinuing the structural wall at various levels. Based upon measured displacements and distributions of story shears between

comprised of moment frames and frame-wall combinations. Irregularities in the vertical frames and walls, it was apparent that the extent of the irregularity could not be gauged solely by comparing the strengths and stiffness's of adjacent stories in a structure. Structures having the same stiffness interruption, but occurring in different stories didn't perform equally.

Moehle and Alarcon (1986) presented a combined experimental and analytical study to examine the seismic response behavior of reinforced concrete frame-wall structures. In one of the models, vertical irregularity in the frame-wall system was introduced by interrupting the wall at the first story level. Inelastic dynamic analysis was capable of adequately reproducing measured displacement waveforms, but accurate matches of responses required a trial and error approach to establish the best modelling assumptions. It was observed that in the vicinity of the discontinuity, the elements exhibited a curvature ductility demand 4 to 5 times higher than in the case of the model without any interruption of the wall.

Costa (1990) extended the previous work (Costa et al. (1988)) on seismic behavior of irregular structures. The study was based on twelve, sixteen, and twenty story reinforced concrete building models. They found the following conclusions: the role of a shear wall in a mixed structural system was to distribute the frame ductilities uniformly along the height, the interruption of a shear wall in part or for the total height of the structure led to a very irregular distribution of frame ductility, also, significant increase was observed in the first level above the interruption of the shear wall. Below the interruption, the behavior was similar to a regular building. In summary it can be observed that analytical and experimental investigations by previous researchers have identified differences in dynamic response of regular and irregular buildings.

Moehle and Alarcon (1986) carried out an experimental response study on two small scale models of reinforced concrete frame-wall structures subjected to strong base motions by using shake table. One of the test structures, designated as FFW, had two nine-story, three-bay frames and a nine-story, prismatic wall. The other structure, designated as FSW, was identical to FFW except that the wall extended only to the first floor level. Thus the test structures FFW and FSW represent the buildings having "regular" and "irregular" distributions of stiffness and strength in vertical plane

respectively. They compared the measured response with that computed by the inelastic dynamic response time-history analysis, inelastic static analysis, elastic modal spectral analysis, and elastic static analysis. Several inelastic response time history analyses were conducted for each test structure. For each analysis, different modeling assumptions were tried in an effort to establish a “best-fit” model. They compared maximum top-floor displacements obtained by the experiments and by different inelastic dynamic and elastic analysis methods.

Ruiz and Diederich (1989) studied the seismic performance of buildings with weak first story in case of single ground motion. They studied the influence of the lateral strength discontinuity on ductility demand at the first story under the action of the acceleration record with largest peak ground acceleration, as obtained on soft soil in Mexico City during the Mexico earthquake of September 19, 1985. A parametric study was carried out for 5- and 12-story buildings with weak first story, and with brittle infill wall in upper stories in some cases and ductile in others. The fundamental periods of these buildings were 0.67 and 1.4 s respectively. They noted that the behavior of weak first story buildings greatly depends on the ratio of the dominant periods of excitation and response, the resistances of upper and first stories, and on the seismic coefficient used for design. The ratio of dominant periods of response and excitation was found to be closely related to the formation of plastic hinges, yielding or failure of infill walls, and to the times of their occurrences.

Esteva (1992) studied the nonlinear seismic response of soft-first-story buildings subjected to narrowband accelerograms. The variables covered were: number of stories, fundamental period, form of the variation of story stiffness along height, ratio of post-yield to initial stiffness, in addition to the variable of primary interest, i.e., factor expressing the ratio of the average value of the safety factor for lateral shear at the upper stories to that at the bottom story. He used shear-beam systems representative of buildings characterized by different number of stories and natural periods as given in Table 2. The study included cases of stories with hysteretic bilinear behavior, both including and neglecting P-delta effects. The excitation was in some cases an accelerogram recorded on soft soil in Mexico City during the Mexico earthquake of September 19, 1985, and in some cases an ensemble of artificial accelerograms with similar statistical characteristics.

He observed that the nature and magnitude of the influence of the ratio on the maximum ductility demands at the first story depend on the low-strain fundamental period of the system. For very short periods those ductility demands may be reduced by about 30% when grows from 1.0 to 3.0. For intermediate periods, ductility demands are little sensitive to  $r$ , but for longer periods those may reach the increments of 50 to 100% while varies within the mentioned interval. He also observed that the influence of on the response of the first story is strongly enhanced if P-delta effects are taken into account.

Valmudsson and Nau (1997) focused on evaluating building code requirements for vertically irregular frames. The earthquake response of 5-, 10-, and 20-story framed structures with uniform mass, stiffness, and strength distributions was evaluated. The structures were modeled as two-dimensional shear buildings. The response calculated from the time-history analysis was compared with that predicted by the ELF procedure as embodied in UBC (1994). Based on this comparison, they evaluated the requirements under which a structure can be considered regular and the ELF provisions are applicable.

Al-Ali and Krawinkler (1998) carried out evaluation of the effects of vertical irregularities by considering height-wise variations of seismic demands. They used a 10-story building model designed according to the strong-beam-weak-column (column hinge model) philosophy and an ensemble of 15 strong ground motions, recorded on rock or firm soil during Western U.S. earthquakes after 1983, for the parametric study. The effects of vertical irregularities in the distributions of mass, stiffness and strength were considered separately and in combinations, and the seismic response of irregular structures was assessed by means of the elastic and inelastic dynamic analyses. They found that the effect of mass irregularity is the smallest, the effect of strength irregularity is larger than the effect of stiffness irregularity, and the effect of combined-stiffness-and-strength irregularity is the largest. Roof displacement is not affected by the vertical irregularity.

Das and Nau (2003) investigated the definition of irregular structure for different vertical irregularities: stiffness, strength, mass, and that dueto the presence of non-structural masonry infill as prescribed in building codes. Linear and nonlinear dynamic time-history (TH) analyses were performed on an ensemble of 78 buildings of 5, 10, and 20 stories and with different story stiffness, strength, and mass ratios. All buildings had three bays in the



direction of the ground motion. The lateral force-resisting systems considered were special moment resisting frames (SMRF) designed based on the forces obtained from the ELF procedure according to the strong-column-weak-beam (SCWB) criteria of ACI 318-99 (ACI, 1999) and UBC (1997). They observed that most structures considered in their study performed well when subjected to the design earthquake ground motion. Hence they concluded that the restrictions on the applicability of the ELF procedure given in building codes are unnecessarily conservative for certain types of vertical irregularities considered

Fragiadakis et al. (2006) proposed a methodology based on Incremental Dynamic Analysis (IDA) to evaluate the response of structures with ‘single-story vertical irregularities’ in stiffness and strength using a nine-story steel frame. IDA is regarded as one of the most powerful analysis methods available, since it can provide accurate estimates of the complete range of the model’s response, from elastic to yielding, then to nonlinear inelastic, and finally to global dynamic instability. IDA involves performing a series of nonlinear dynamic analyses for each record by scaling it to several levels of intensity.

Moehle and Alarcon (1986) carried out an experimental response study on two small scale models of reinforced concrete frame-wall structures subjected to strong base motions by using shake table. One of the test structures, designated as FFW, had two nine-story, three-bay frames and a nine-story, prismatic wall. The other structure, designated as FSW, was identical to FFW except that the wall extended only to the first floor level. Thus the test structures FFW and FSW represent the buildings having “regular” and “irregular” distributions of stiffness and strength in vertical plane respectively. They compared the measured response with that computed by the inelastic dynamic response time-history analysis, inelastic static analysis, elastic modal spectral analysis, and elastic static analysis. Several inelastic response time history analyses were conducted for each test structure. For each analysis, different modeling assumptions were tried in an effort to establish a “best-fit” model. They compared maximum top-floor displacements obtained by the experiments and by different inelastic dynamic and elastic analysis methods. They concluded that the main advantage of dynamic methods is that those are capable of estimating the maximum displacement response, whereas the static

methods cannot be used for this purpose. Further, they inferred that the inelastic static and dynamic methods are superior to the elastic methods in interpreting the structural discontinuities.

Ruiz and Diederich (1989) studied the influence of the lateral strength discontinuity on ductility demand at the first story under the action of the acceleration record with largest peak ground acceleration, as obtained on soft soil in Mexico City during the Mexico earthquake of September 19, 1985. A parametric study was carried out for 5- and 12-story buildings with weak first story, and with brittle infill wall in upper stories in some cases and ductile in others. The fundamental periods of these buildings were 0.67 and 1.4 s respectively. They noted that the behavior of weak first story buildings greatly depends on the ratio of the dominant periods of excitation and response, the resistances of upper and first stories, and on the seismic coefficient used for design. The ratio of dominant periods of response and excitation was found to be closely related to the formation of plastic hinges, yielding or failure of infill walls, and to the times of their occurrences.

## 1.6 Overview from Literature Review

The seismic response of vertically irregular building frames, which has been the subject of numerous research studies, started getting attention in the late 1970s. A large number of studies have focused on plan irregularity resulting in torsion in structural systems. Vertical irregularities are characterized by vertical discontinuities in the distribution of mass, stiffness and strength. Very few research studies have been carried out to evaluate the effects of discontinuities in each one of these quantities independently, and majority of the studies have focused on the elastic response. There have also been detailed studies on real irregular buildings that failed during earthquakes. Studies aimed to predict the behavior of structures with vertical irregularities are small in number compared to the studies aimed to predict the behavior of structures with horizontal irregularity. Nevertheless, in recent years research activity in this field has been growing. Researchers have a lot of studies for the effects of vertical irregularities on the seismic behavior of

structures. These irregularities are characterized by vertical discontinuities in the distributions of masses, stiffness's, and strengths.

### 1.7 Objectives and Scope of the Study

Stress variation in regular and irregular structures has been observed. The goal has been set to perform finite element analysis by ABAQUS software through which stress distribution in joints of regular and irregular structure has been differentiated. Measurement of the vulnerability of the structure due to earthquake load and wind load through finite element analysis could be done. Parking facility in the ground floor due to vertical discontinuity can be enhanced. Determination of stress distribution can further result in structural health monitoring.

### 1.8 Organization of Thesis

In this thesis, we have analyzed two cases. For this purpose, we have analyzed the cases both numerically. For numerical analysis we have used a software named ABAQUS v6 10.1. Comparison is done between two cases to determine the stress distribution at joints. This thesis represents the comparative study in-between the regular structure and irregular structure.

**CHAPTER TWO**  
**FINITE ELEMENT ANALYSIS**

## 2.1. General

Two cases involving one regular and one irregular frame have been analyzed. For numerical analysis, we have used a software named “ABAQUS 6.10, where the effects of vertical and horizontal loads are considered. Displacement/Rotation -type boundary condition is used in both cases. Pressure-type load condition is used in both cases. Loads have been calculated from a typical 2- storied building.

## 2.2 Finite Element Analysis

Finite element method (FEM) is a numerical technique for finding approximate solutions to boundary value problems for partial differential equations. It uses various methods (the calculus of variations) to minimize an error function and produce a stable solution. Analogous to the idea that connecting many tiny straight lines can approximate a larger circle, FEM encompasses all the methods for connecting many simple element equations over many small subdomains, named finite elements, to approximate a the more complex equation over a larger domain.

The diagram consists of two yellow rectangular boxes with orange borders. The top box contains the text "Governing Equation (Differential equation)" and the equation  $L(\phi) + f = 0$ . Below this box is a large orange plus sign. The bottom box contains the text "Boundary Conditions" and the equation  $B(\phi) + g = 0$ .

$$\begin{array}{c} \text{Governing Equation} \\ \text{(Differential equation)} \\ L(\phi) + f = 0 \\ + \\ \text{Boundary Conditions} \\ B(\phi) + g = 0 \end{array}$$

Figure 2.1 Finite element equation

## How does FEA work?

Two key steps in numerical integration:

1. Divide the interval of integration.
2. In each sub-interval, choose proper simple functions to approximate the true function.

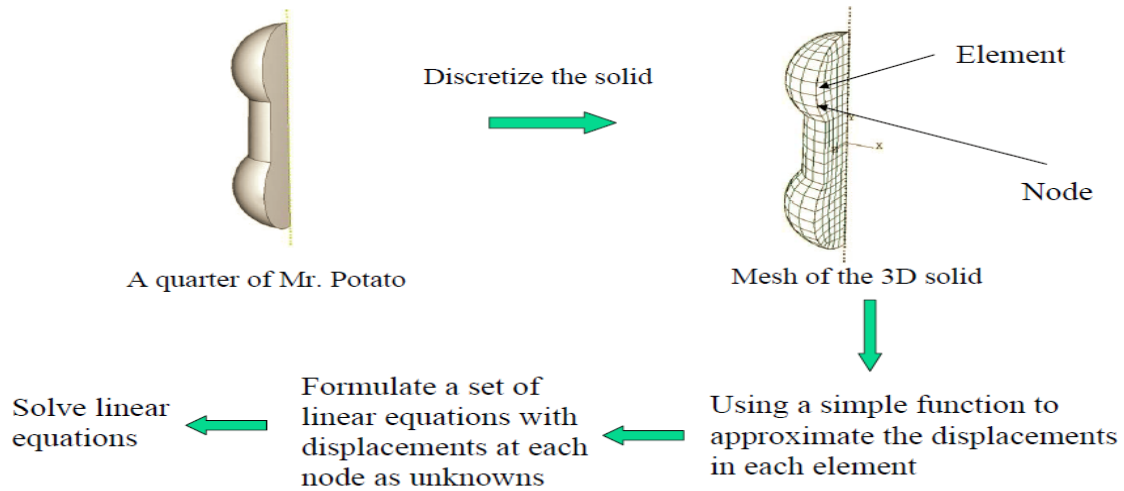


Figure 2.2 Finite element analysis

## 2.3 Material Modelling

### 2.3.1 Concrete Damage Plasticity

Given that concrete displays the characteristics of both a plastic material and a damaging material, it is appropriate to develop models that incorporate both mechanisms of response. In recent years, two types of elastic-plastic-damage models have been proposed.

Several of these models are developed on the basis of plasticity theory and the assumption that material damage appropriately is defined by the accumulated plastic strain. The model proposed by Lubliner et al. [1989] has the following characteristics:

- The shape of the yield surface is assumed to remain constant and is defined by a modified Mohr-Coulomb criterion.
- The evolution of the elastic domain is defined by a hardening rule that is calibrated on the basis of experimental data.
- Plastic strain is defined on the basis of an associated flow rule.

- Damage is assumed to be isotropic and defined by a single scalar damage variable “k” that is a measure of the accumulated damage.
- Damage is assumed to accumulate as a function of plastic strain

Model proposed by Govindjee and Hall [1997] considers a damage model to characterize the response of concrete in tension and shear and a plastic model to characterize the response of concrete in compression. Additionally, this model has the following characteristics:

- Anisotropic damage model with the orientation of damage established by formation of a single fixed fictitious crack surface that is perpendicular to the direction of the peak principal tensile stress.
- The damage/failure surface defines an undamaged concrete tensile strength and shear strength; damage initiates when the trial principal tensile strength exceeds to concrete tensile strength.
- The damage surface has an exponential softening rule with accumulated damage occurring through tensile and shear action on the fictitious crack surface.
- Single surface plasticity model with associated plastic flow.

This model has the advantages of the anisotropic, crack-oriented damage models previously identified. Additionally, the partial decoupling of the damage and plasticity modes of response provides enhanced numerical efficiency by allowing for consideration of only a single mode of response for appropriate trial stress states. It is possible for both the plasticity and damages surfaces to be active for a particular strain increment.

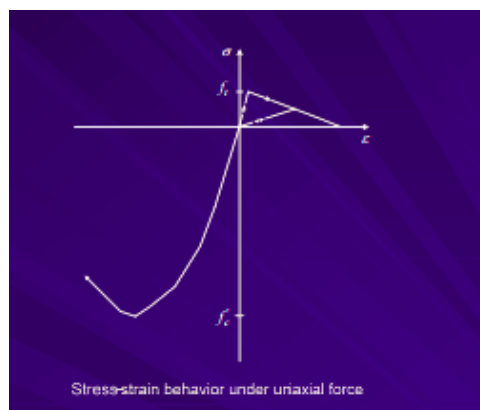


Figure: 2.3 Concrete Damage Plasticity

### 2.3.2 Linear Elastic Material Model

Steel of the reinforcing bars has approximately linear elastic behavior when the steel stiffness introduced by the Young's or elastic modulus keeps constant at low strain magnitudes. Once the stress in the steel exceed the yield stress, permanent (plastic) deformation begins to occur. Both elastic and plastic strains accumulate as the metal deforms in the post-yielding region. The stiffness of the steel decreases once the material yields. The plastic deformation of the steel material increases its yield stress for subsequent loadings. Tie bar spacing is 8" and hook length is 6".

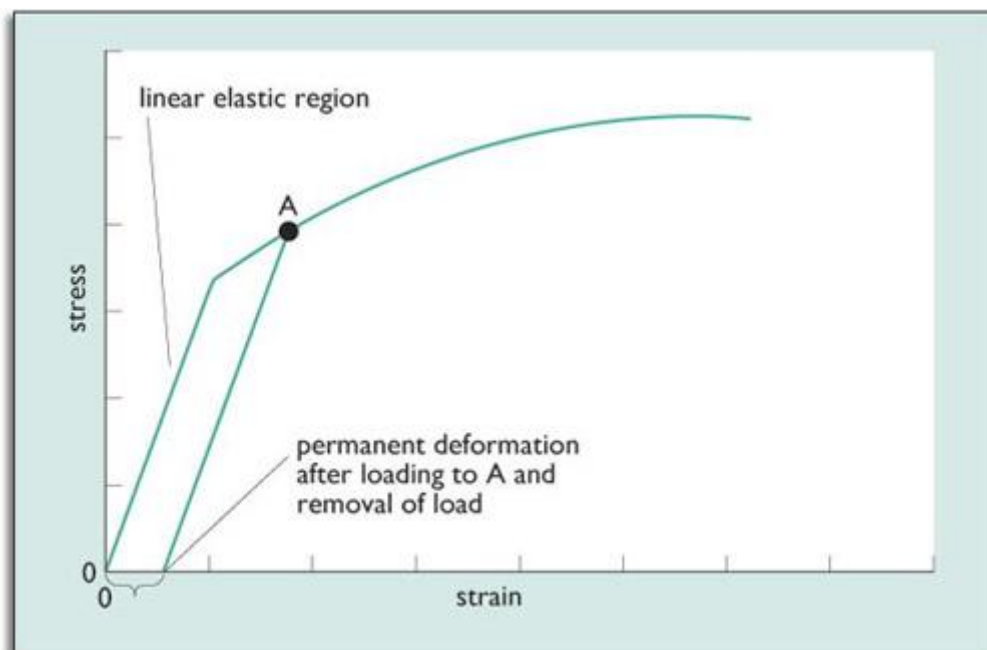


Figure 2.4: Linear elastic model



## 2.4 Embedded Element

The embedded element technique is used to specify that an element or group of elements is embedded in “host” elements. The embedded element technique can be used to model rebar reinforcement. Abaqus searches for the geometric relationships between nodes of the embedded elements and the host elements. If a node of an embedded element lies within a host element, the translational degrees of freedom at the node are eliminated and the node becomes an “embedded node.” The translational degrees of freedom of the embedded node are constrained to the interpolated values of the corresponding degrees of freedom of the host element. Embedded elements are allowed to have rotational degrees of freedom, but these rotations are not constrained by the embedding. Multiple embedded element definitions are allowed. We have used truss-in-solid element type for our modelling.. By default, the elements in the vicinity of the embedded elements are searched for elements that contain embedded nodes; the embedded nodes are then constrained by the response of these host elements. To preclude certain elements from constraining the embedded nodes. This feature is strongly recommended if the embedded nodes are close to discontinuities in the model (cracks, contact pairs, etc.). The drawbacks of using embedded element are:

- Elements with rotational degrees of freedom (except axisymmetric elements with twist) cannot be used as host elements.
- Rotational, temperature, pore pressure, acoustic pressure, and electrical potential degrees of freedom at an embedded node are not constrained.
- Host elements cannot be embedded themselves.
- The material defined for the host element is not replaced by the material defined for the embedded element at the same location of the integration point.

Additional mass and stiffness due to the embedded elements are added to the model.

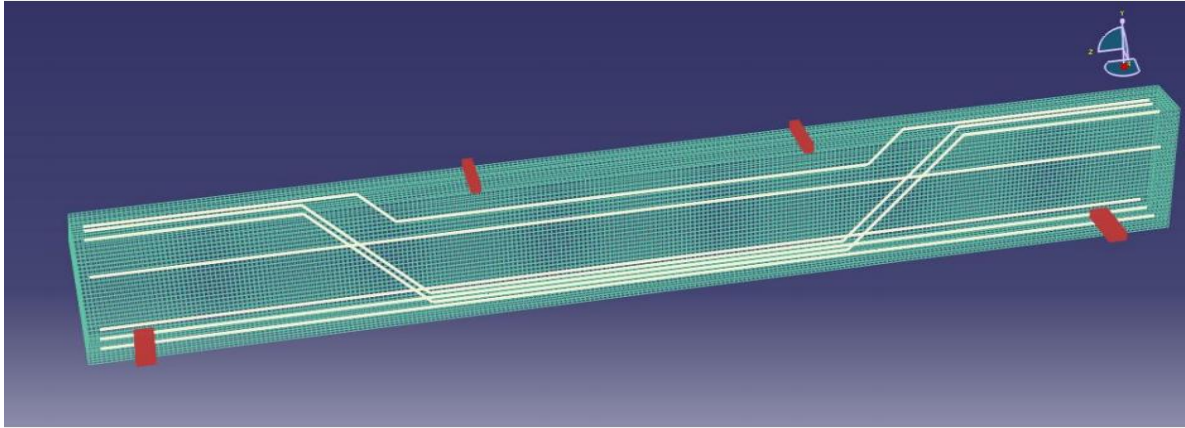


Figure 2.5 embedded element

## 2.5 Boundary Condition

A beam element accounting for bond-slip has 4 degrees of freedom in each node, horizontal translation (x-direction), vertical translation (z-direction), rotation around (y-axis) and slip in the interface between concrete and steel. That means for the simplest case a straight beam supported in two nodes at the ends needs 8 boundary conditions essential, natural or convective. Convective boundary conditions are a combination of essential and natural. Table 3.1: Possible essential (Dirichlet) and natural (Neumann) boundary conditions for the reinforced beam element accounting for embedded panel. In Table 3.1 the possible boundary conditions are listed. However there are certain limitations to what boundary conditions combinations give unique solutions. To avoid rigid body motion both the criteria below need to be fulfilled:

- At least one degree of freedom in the horizontal direction has to be prescribed.
- One vertical translational degree of freedom and one rotational or two vertical translational

Degrees of freedom need to be prescribed Displacement/ Rotation type boundary condition has been implemented here. Displacement/rotation boundary condition is used to constrain the movement of the selected degrees of freedom to zero or to prescribe the displacement or rotation for each selected degree of freedom.

## 2.6 Loading Conditions

Usually some form of external loading is defined. For example, concentrated or distributed loads can be applied, temperature changes leading to thermal expansion can be prescribed, or contact conditions can be used to apply loads the loading can be prescribed as a function of time. This feature can be used to prescribe loadings such as the ground motion during a seismic event, known accelerations, or the temperature and pressure history during a transient in an engine. If an amplitude curve is not defined, ABAQUS assumes either that the loading varies linearly over the step or that the load is applied instantaneously at the beginning of the step, depending on the chosen response type.

## 2.7 Finite Element Software

ABAQUS/CAE is capable of pre-processing, post-processing, and monitoring the processing stage of the solver.

Pre-processing or modeling: This stage involves creating an input file which contains an engineer's design for a finite-element analyzer (also called "solver").

Processing or finite element analysis: This stage produces an output visual file.

Post-processing or generating report, image, animation, etc. from the output file: This stage is a visual rendering stage.

## 2.8 Selection of ABAQUS to Perform Finite Element Analysis

This software is user friendly and make element-type understanding easier. ABAQUS was conceived as a non-linear solver that handles linear models as a particular case. This feature makes it extremely powerful, robust and easy to use moving from one field to the other one just one click away of geometrical no-linear or material models. Handling of

parts instances into assemblies, material models available, non-linear analysis capabilities, integration of implicit and explicit codes, contact robustness are definite in this software.

## 2.9 Load Calculation

Structural members must always be proportioned to resist loads greater than service or actual loads, in order to provide proper safety against failure. In the strength design method, the member is designed to resist the factored loads which are obtained by multiplying the factored loads with live loads. Total distributed load provided is 36kips/ft.

Different factors are used for different loadings. As dead loads can be estimated quite accurately, their load factors are smaller than those of live loads, which have a high degree of uncertainty. Several load factor conditions must be considered in the design to compute the maximum and minimum design forces. Reduction factors are used for some combinations of loads to reflect the low probability of their simultaneous occurrences. Now if the ultimate load is denoted by U, the according to the ACI code, the ultimate required strength U, shall be the most critical of the following:

Basic Equation  $U = 1.2D + 1.6L$

Wind Load Calculation Formulae according to UBC'97

Wind Load calculated= 14.4 kips/ft.

Force=  $A \times P$

A = the projected area of the item.

$P = \text{Wind pressure (Psf)} = C_e \times C_q \times Q_s$

$C_e$ = combined height, exposure and gust response factor

$C_q$ =pressure coefficient (same as drag,  $C_d$ )

$C_q = 1.3$  for flat plates, and  $C_q = .8$  for cylinders over 2" in diameter, 1.0 for cylinders 2" or less in diameter.

$Q_s$ =wind stagnation pressure.

## CHAPTER THREE

### **METHODOLOGY**

### 3.1 General

In this thesis paper we have considered two cases. One is a regular frame structure and the other is irregular frame structure. Similar load condition and boundary conditions are provided for both cases.

### 3.2 Pre Processing

#### 3.2.1 Frame Modelling

The concrete frame of a two-storied model has been drawn. Concrete damage plasticity type material model has been used here. Co-ordinate system has been used to draw the frame. Section assignment has been done for the frame. The frame acted as host element for the reinforcement provided.

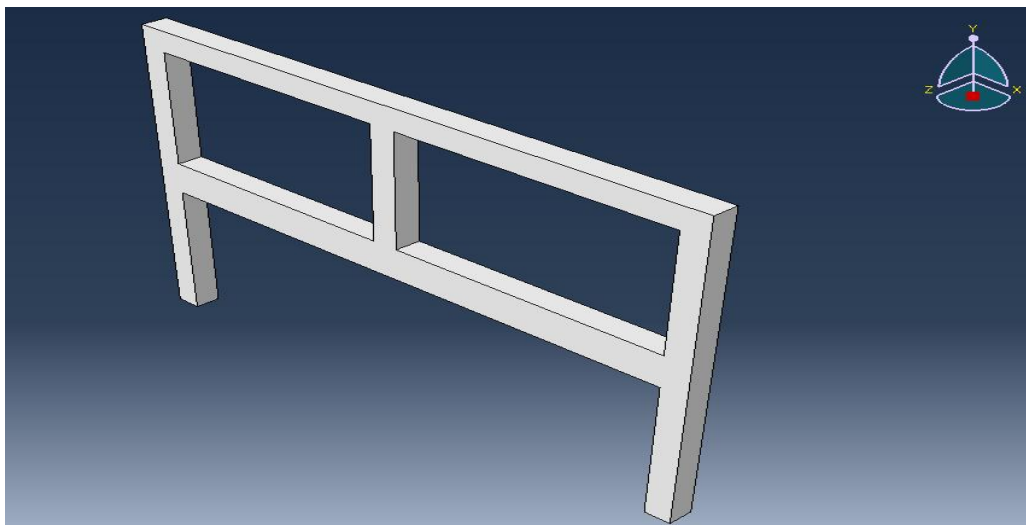


Figure 3.1: Concrete Frame Modelling

#### 3.2.2 Reinforcement Modelling

Reinforcement modelling has been done for beam and column respectively. Linear-elastic element type has been used. Co-ordinate system has been used to draw the frame. Section assignment has been done for the reinforcement. The reinforcement used as embedded element here.

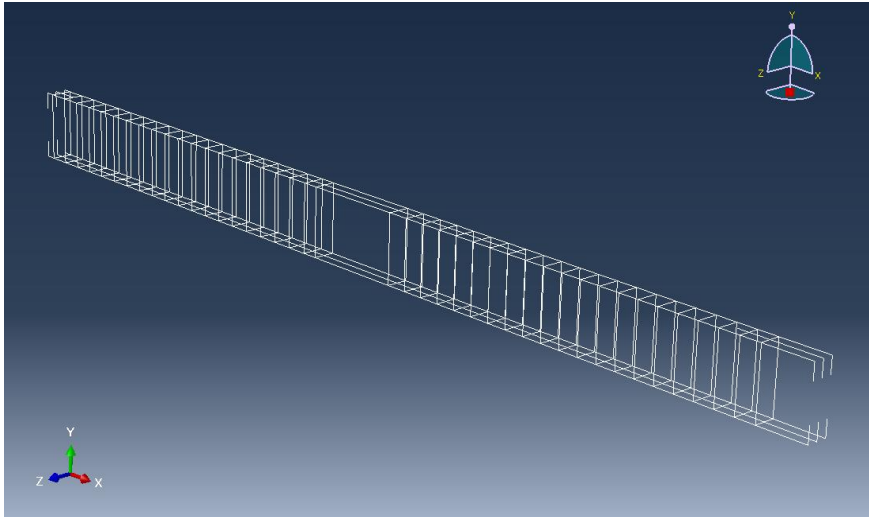


Figure 3.2 Beam Reinforcement

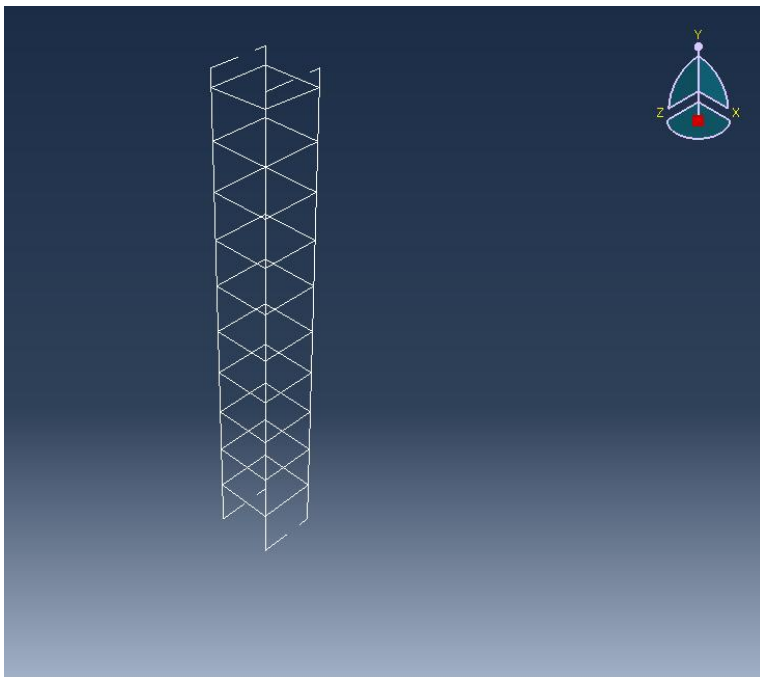


Figure 3.3 Column Reinforcement



### 3.3 Case 1

In 1<sup>st</sup> case we considered a regular frame structure. Beam size is placed as 20"x18", grade beam as 30"x18" and column size as 18"x18". Middle column reinforcement and girder reinforcement have been inserted. Column Spacing of 17.5" is used. Wind load equal to 14.4kips/ft. has been inserted on windward direction. Distributed beam load of 24kips/ft. has been inserted.

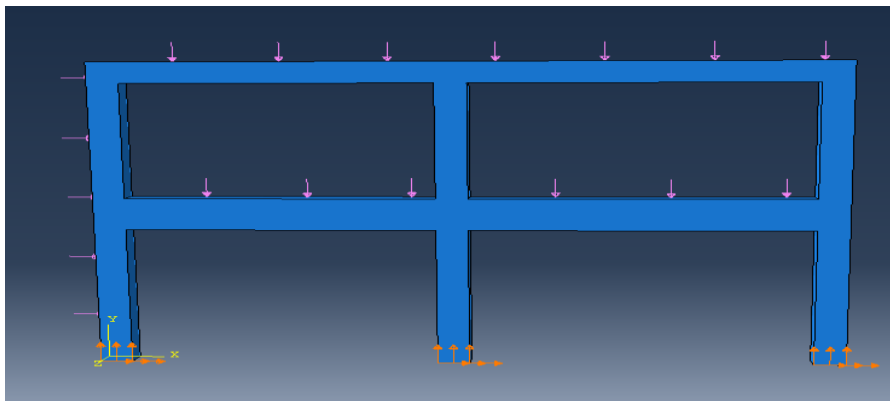


Figure 3.4 Application of horizontal and vertical load on regular frame structure.

### 3.4 Case 2

In the second case, we have removed the middle column in the ground floor. Dimensions are kept similar to case 1. Load and boundary conditions are also kept alike as case 1.

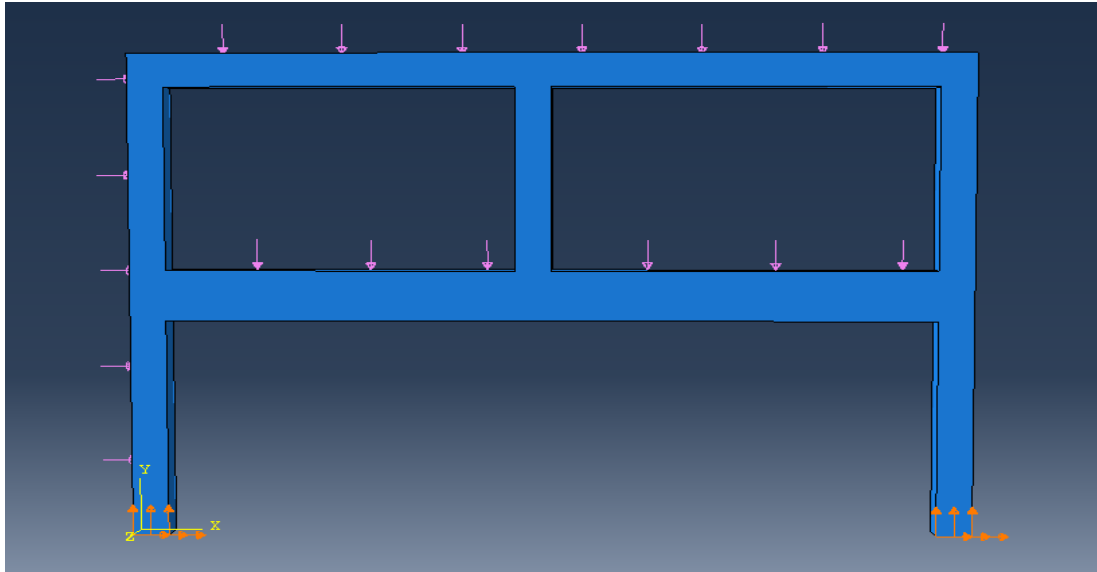


Figure 1.5 Application of horizontal and vertical load on irregular frame structure.

### 3.4 Material Properties of Concrete Beam

Material properties for all elements have been specified. Material Damage Plasticity has been used. Density used here equals to  $0.87\text{lb/in}^3$ , Young Modulus is  $2.17\text{e}+07$ , and Poisson's ratio 0.25. Dilation angle  $36.37$ , eccentricity 0,  $f_{b0}/f_{c0}$  is 1.11,  $K$  0.67, Viscosity Parameter 0.002. Here concrete compression hardening and concrete tension stiffening are used. The validity of the results has been essentially limited by the accuracy and extent of the material data. Here linear elasticity and on linear plasticity of reinforcement steel and isotropic elasticity in combination with damaged plasticity model of concrete has been discussed to depict a mechanical constitutive model. Element used here were C3D8R which is three-dimensional hexahedral element. Tetrahedral elements are geometrically versatile and are used in many automatic meshing algorithms. However, a good mesh of hexahedral elements (C3D8R) usually provides a solution of equivalent accuracy at less cost. First-order tetrahedral and triangles are usually overly stiff, and extremely fine meshes are required to obtain accurate results.

### 3.5 Material Properties of Reinforcement bar

T3D2 element has been used. Truss elements are rods that can carry only tensile or compressive loads. They have no resistance to bending; therefore, they are useful for modeling pin-jointed frames. When a beam is very slender, it can be modeled as a truss. The yield and ultimate strengths were taken for longitudinal steel bars 379 MPa and 581 MPa and confinement steel bars 373 MPa and 518 MPa with Young's modulus of elasticity, with Young's modulus of elasticity,  $E_s = 2.9 \times 10^7$  psi and Poisson's ratio = 0.3.

**CHAPTER FOUR**  
**RESULT AND DISCUSSION**

## 4.1 General

Comparative study between the two cases has been done here. Von Mises stress, Tresca stress and Maximum Principle Stress have been considered as point of analysis and comparison. The maximum and minimum stresses are observed in definite nodes and element. Allowable maximum and minimum stresses have been assigned for our study. The values of stresses exceeding the assigned allowable maximum and minimum stresses are considered as failure zones.

- Allowable Maximum Stress =  $1.579e^2$  kips/inches<sup>2</sup>
- Allowable Maximum Stress =  $1.579e^2$  kips/inches<sup>2</sup>
- Allowable Minimum Stress =  $2.372e-2$  kips/inches<sup>2</sup>
- Allowable Stress = Material Strength / Factor of Safety
- Design is satisfactory when,  $R_a \leq R_n / \Omega$  where  $R_a$  is required strength of material,  $R_n$  is nominal strength,  $\Omega$  is factor of safety  $> 1$
- Failure Zone exists where Maximum Stress  $>$  Allowable Stress

## 4.2 Comparative Study of Von Mises Stress

### 4.2.1 Case 1

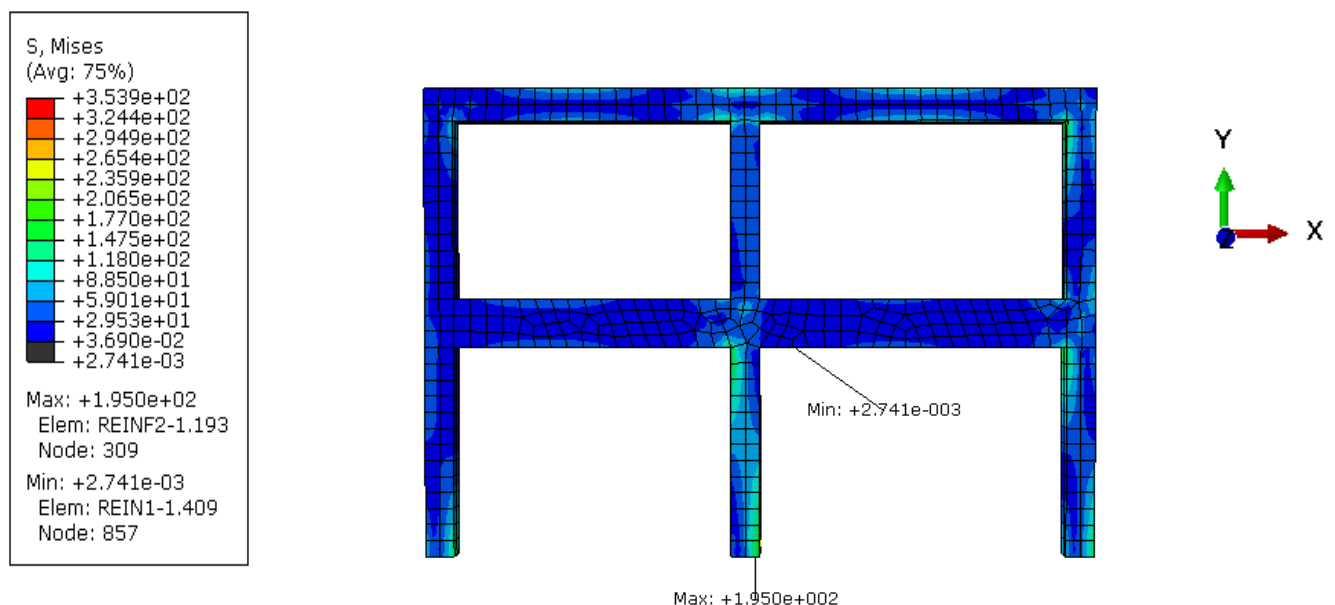


Figure 4.1 Von Mises Stress distribution in regular structure

#### 4.2.2 Case 2

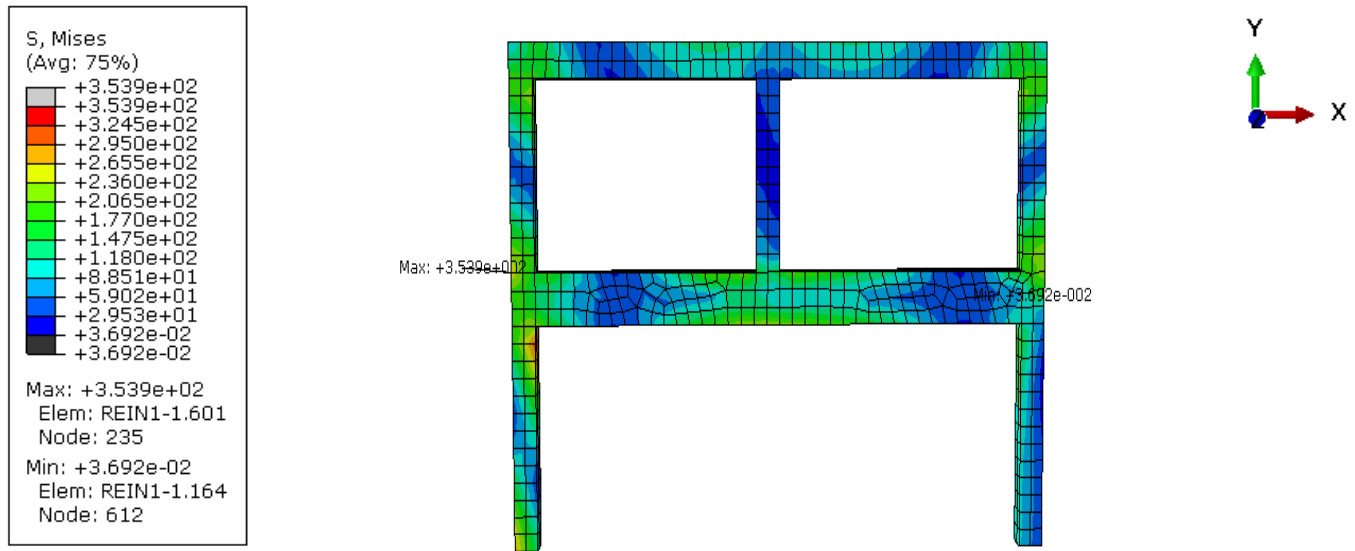


Figure 4.2. Von Mises Stress distribution in irregular structure

For Von mises stress, reddish areas exceed allowable maximum stress which indicates that these zones are vulnerable to failure. In case 1 red zoned areas are fewer than case 2 which indicates that the frame structure is more stable. In Case 2 maximum red zones are found in beam column joints. Hence emphasis should be given on beam column joints for vertical discontinuous structures.

### 4.3 Comparative Study of Tresca Stress

#### 4.3.1 Case 1

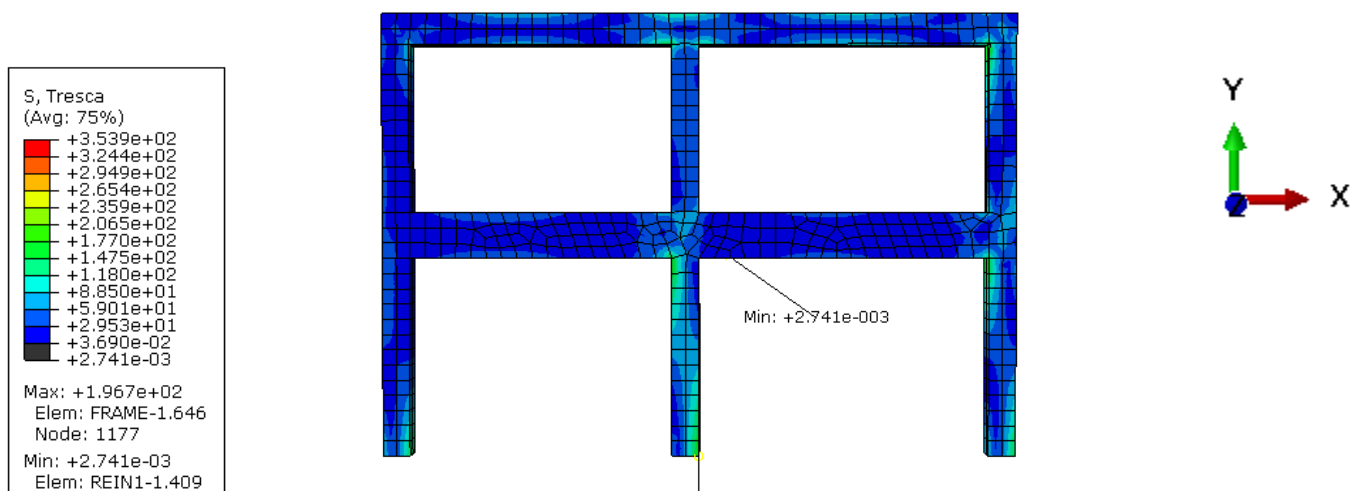


Figure 4.3. Tresca Stress distribution in regular structure

### 4.3.2 Case 2

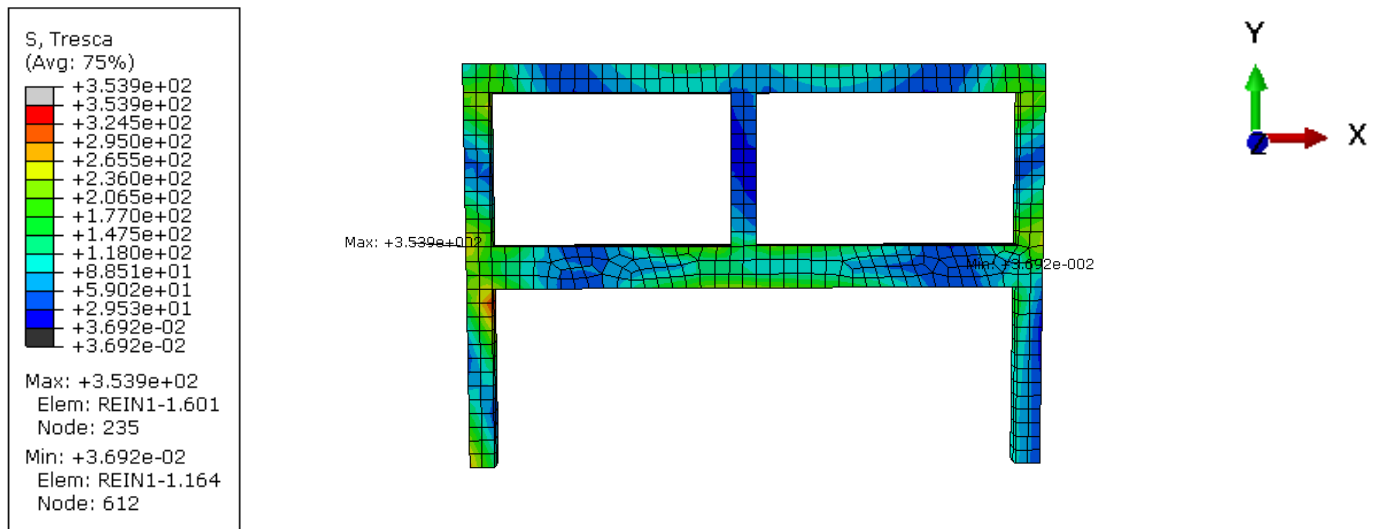


Figure 4.3.2 Tresca Stress distribution in irregular structure

For Tresca stress, a reddish or yellowish area exceeds allowable maximum stress which indicates that these zones are vulnerable to failure. In case 1 reddish or yellowish areas are fewer than case 2 which indicates that the frame structure is more stable. In Case 2 maximum reddish or yellowish zones are found in beam column joints. Hence emphasis should be given on beam column joints for vertical discontinuous structures.

## 4.4 Comparative Study of Maximum Principle Stress

### 4.4.1 Case 1

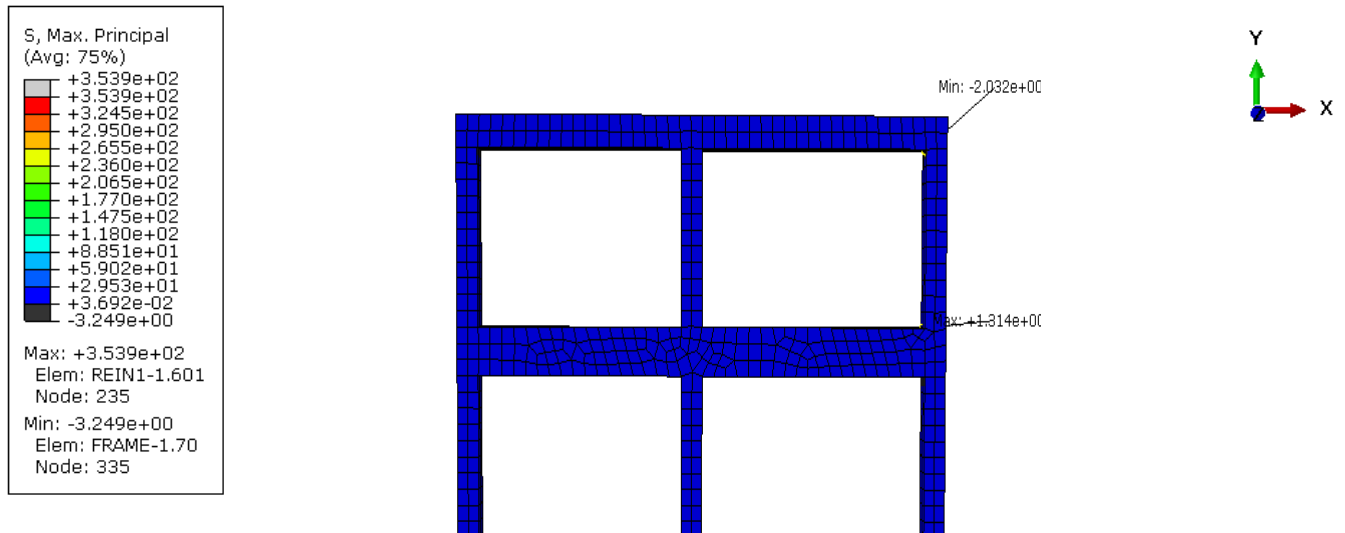


Figure 4.4.1 Maximum Principle Stress distribution in regular structure



#### 4.4.2 Case 2

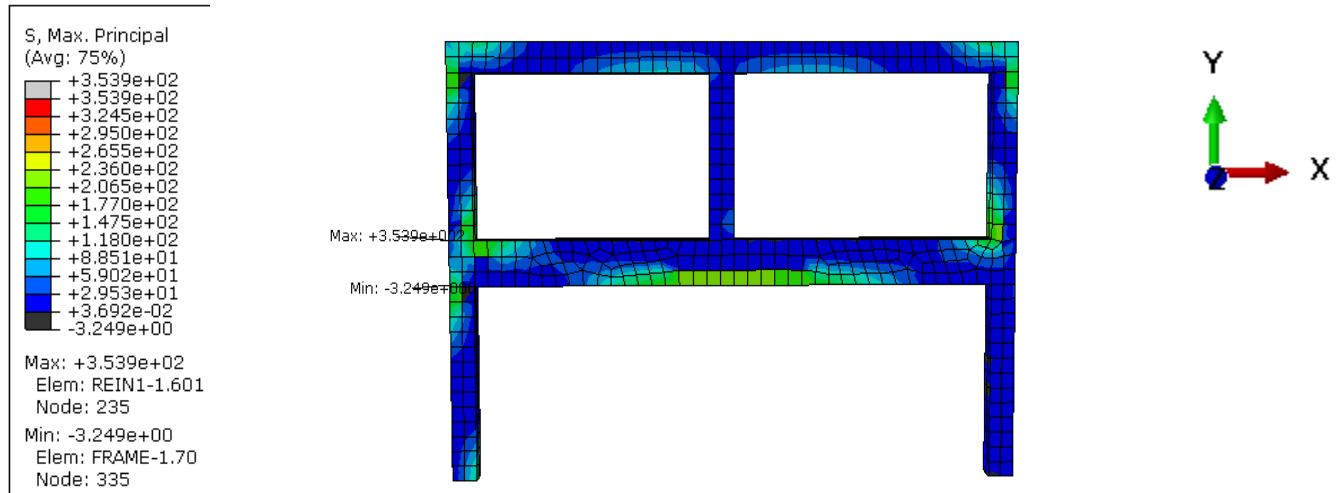


Figure 4.4.2 Maximum Principle Stress distribution in irregular structure

For Maximum Principle stress, yellowish zone exists very slightly in case 2 and there is no significant effect of stress distribution in case 1. Hence vulnerability in terms of maximum principle stress is very much limited.

It has been observed here that the difference in stress distribution in both cases varied in a significant amount. The simulation indicates that the irregular structure frame is more vulnerable.

## **CHAPTER FIVE**

### **CONCLUSION AND RECOMMENDATION**

## 5.1 General

For this analysis, modeling of frame has been done by using ABAQUS software. The analysis have been done here by using concrete damage plasticity in case of concrete material modeling and we have used linear elastic modeling in case of reinforcement modeling. The reinforcement was embedded with full interaction between concrete and steel.

## 5.2 Comparison of maximum and minimum values of different stresses

Name of the Stress	Case 1		Case2		%change in stress distribution	
	Maximum stress	Minimum stress	Maximum stress	Minimum stress	Maximum Stress	Minimum Stress
Von Mises	$1.95 \times 10^2$ (node 309)	$2.741 \times 10^{-3}$ (node 857)	$3.539 \times 10^2$ (node 235)	$3.692 \times 10^{-2}$ (node 612)	81.4%	92.5%
Tresca	$1.967 \times 10^2$ (node 1177)	$2.741 \times 10^{-3}$ (node 857)	$3.539 \times 10^2$ (node 235)	$3.692 \times 10^{-2}$ (node 612)	79.92%	92.5%
Maximum Principle Stress	$3.539 \times 10^2$ (node 235)	-3.249 (node 335)	$3.539 \times 10^2$ (node 235)	-3.249 (node 335)	0%	0%

It has been observed that stress distribution in case 2 is very high with respect to case 1. In case of Von mises stress, stress distribution has been increased by 1.814 times in irregular structure in comparison with regular structure. Similarly Tresca stress has been increased by 1.799 times in irregular structure in comparison with regular structure. But no significant change is observed in case of Maximum Principle stress.

### 5.3 Recommendation and Conclusion

In this model, deflection and crack has been analyzed for regular and irregular structure. It has been observed from the obtained result that the maximum stresses (Von Mises, Tresca, and Maximum Principle Stresses) are comparatively lower in regular frame structure than the irregular one. The nodes at which maximum stresses occur at irregular frame are mostly lie on beam column joints. This suggests that these joints are susceptible to damage and crack formation. These joints are precisely analyzed by finite element analysis which provides a better result than experimental analysis or non-finite element analysis. Through stress analyses of different structures subjected to vertical discontinuity we can select a structure less susceptible to crack and damage formation.

The nonlinear finite element model was developed to simulate stress analysis of a regular frame structure and an irregular structure. This paper presents a finite element model which can be used to analyze the non-linear behavior of reinforced concrete elements. This paper compares the numerical results of two different structures subjected to horizontal and flexural loading. Recommended measures to minimize effect of increased irregular structure are as follows:

- Length of grade beam can be decreased
- Usage of resisting frame structure
- Stiffness of grade beam can be increased
- Column size can be increased
- High strength material can be used

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